Quiz 9, Math 220, Professor David Levermore
Wednesday, 27 October 2010

(1) [3] Find the first derivative of $g(z) = \ln(1 + e^{\pi z})$.

Solution. By the logorithm and exponential rules for differentiation

$$g'(z) = \frac{1}{1 + e^{\pi z}} \frac{\mathrm{d}}{\mathrm{d}z} e^{\pi z} = \frac{1}{1 + e^{\pi z}} e^{\pi z} \pi \,.$$

(2) [3] Solve $\ln\big(\ln(2y)\big) = 0$ for y.

Solution. Exponentiate both sides of the given equation to get

$$\ln(2y) = e^{\ln(\ln(2y))} = e^0 = 1 \,.$$

Next, exponentiate both sides of this equation to get

$$2y = e^{\ln(2y)} = e^1 = e \,.$$

Hence, $y = e/2$.

(3) [4] Find the first derivative of $f(x) = \ln\big((x+1)^2(2x+1)^3(x+7)^5\big)$.

Solution. By the laws of logorithms you see that

$$\begin{aligned} f(x) &= \ln\big((x+1)^2\big) + \ln\big((2x+1)^3\big) + \ln\big((x+7)^5\big) \\ &= 2\ln(x+1) + 3\ln(2x+1) + 5\ln(x+7) \,. \end{aligned}$$

By the logorithm rule for differentiation

$$f'(x) = 2\frac{1}{x+1} + 3\frac{2}{2x+1} + 5\frac{1}{x+7} = \frac{2}{x+1} + \frac{6}{2x+1} + \frac{5}{x+7} \,.$$

Remark. This approach is related to logrithmic differentiation.

Alternative Solution. By the logorithm, product, and power rules of differentiation

$$\begin{aligned} f'(x) &= \frac{1}{(x+1)^2(2x+1)^3(x+7)^5} \frac{\mathrm{d}}{\mathrm{d}x}\big((x+1)^2(2x+1)^3(x+7)^5\big) \\ &= \frac{1}{(x+1)^2(2x+1)^3(x+7)^5} \\ &\quad \cdot \Big(2(x+1)(2x+1)^3(x+7)^5 + (x+1)^2 3(2x+1)^2 2(x+7)^5 \\ &\qquad\qquad + (x+1)^2(2x+1)^3 5(x+7)^4\Big) \\ &= \frac{2}{x+1} + \frac{6}{2x+1} + \frac{5}{x+7} \,. \end{aligned}$$