Quiz 11, Math 220, Professor David Levermore
Friday, 12 November 2010

(1) [5] The number of people q willing to ride a regional train when its tickets are priced at p dollars is $q = 800(4 - \sqrt{p})$. Currently tickets are priced at 9 dollars.

(a) Is the demand elastic or inelastic when $p = 9$?

(b) Should the price of a ticket be raised or lowered to increase revenue?

Solution. The elasticity of a demand $q = f(p)$ is given by

$$E(p) = -\frac{p\, f'(p)}{f(p)}\,.$$

For $f(p) = 800(4 - \sqrt{p}) = 800(4 - p^{\frac{1}{2}})$ one has $f'(p) = 800(-\frac{1}{2}p^{-\frac{1}{2}})$, so that

$$E(p) = -\frac{p\, 800(-\frac{1}{2}p^{-\frac{1}{2}})}{800(4 - p^{\frac{1}{2}})} = \frac{\frac{1}{2}p^{\frac{1}{2}}}{4 - p^{\frac{1}{2}}}\,.$$

Because $9^{\frac{1}{2}} = 3$ we see that

$$E(9) = \frac{\frac{1}{2}9^{\frac{1}{2}}}{4 - 9^{\frac{1}{2}}} = \frac{\frac{1}{2}3}{4 - 3} = \frac{\frac{3}{2}}{1} = \frac{3}{2}\,.$$

(a) Because $E(9) = \frac{3}{2} > 1$ the demand is *elastic* when $p = 9$.

(b) Because the demand is elastic, the price should be *lowered* to increase revenue.

(2) [5] Determine $\displaystyle\int \left(e^{2x} - 5x^3 + \frac{3}{x}\right) \mathrm{d}x$

Solution. Because

$$\int e^{2x}\,\mathrm{d}x = \frac{1}{2}\,e^{2x} + C\,, \qquad \int x^3\,\mathrm{d}x = \frac{1}{4}\,x^4 + C\,, \qquad \int \frac{1}{x}\,\mathrm{d}x = \ln(|x|) + C\,,$$

you see that

$$\int \left(e^{2x} - 5x^3 + \frac{3}{x}\right) \mathrm{d}x = \frac{1}{2}\,e^{2x} - 5\,\frac{1}{4}\,x^4 + 3\ln(|x|) + C\,.$$