Quiz 12 Solutions, Math 220, Professor David Levermore
Friday, 19 November 2010

(1) [5] Evaluate the definite integral $\int_0^1 \left(e^{-x} + x^{\frac{1}{3}}\right) \mathrm{d}x$.

Solution. Because $-e^{-x}$ is an antiderivative of e^{-x} and $\frac{3}{4}x^{\frac{4}{3}}$ is an antiderivative of $x^{\frac{1}{3}}$, by the Fundamental Theorem of Calculus

$$\begin{aligned}\int_0^1 \left(e^{-x} + x^{\frac{1}{3}}\right) \mathrm{d}x &= \left(-e^{-x} + \tfrac{3}{4}x^{\frac{4}{3}}\right)\Big|_0^1 \\ &= \left(-e^{-1} + \tfrac{3}{4}\right) - \left(-e^0 + 0\right) \\ &= -\frac{1}{e} + \tfrac{3}{4} + 1 = \tfrac{7}{4} - \frac{1}{e}\,.\end{aligned}$$

(2) [5] Compute the area of the region bounded by the curves $y = x^2$ and $y = x + 2$.

Solution. The curves intersect when $x^2 = x + 2$. By solving for x you find

$$0 = x^2 - x - 2 = (x+1)(x-2)\,,$$

which means that the curves intersect when $x = -1$ and $x = 2$. By checking $x = 0$ you see that the curve $y = x + 2$ lies above the curve $y = x^2$. The area of the region bounded by the curves $y = x^2$ and $y = x + 2$ is given by

$$\begin{aligned}\int_{-1}^2 \left(x + 2 - x^2\right) \mathrm{d}x &= \left(\tfrac{1}{2}x^2 + 2x - \tfrac{1}{3}x^3\right)\Big|_{-1}^2 \\ &= \left(\tfrac{1}{2}2^2 + 4 - \tfrac{1}{3}2^3\right) - \left(\tfrac{1}{2} - 2 + \tfrac{1}{3}\right) \\ &= 2 + 4 - \tfrac{8}{3} - \tfrac{1}{2} + 2 - \tfrac{1}{3} = \tfrac{9}{2}\,.\end{aligned}$$