

Quiz 13 Solutions, Math 220, Professor David Levermore
Wednesday, 24 November 2010

- (1) [6] Compute the average value of $r(x) = 6x^{\frac{1}{2}}$ over the interval from $x = 1$ to $x = 9$. Simplify your answer.

Solution. The average value is given by

$$\frac{1}{9-1} \int_1^9 6x^{\frac{1}{2}} dx.$$

Because $4x^{\frac{3}{2}}$ is an antiderivative of $6x^{\frac{1}{2}}$, you find that

$$\begin{aligned} \frac{1}{9-1} \int_1^9 6x^{\frac{1}{2}} dx &= \frac{1}{8} \left(4x^{\frac{3}{2}} \right) \Big|_1^9 = \frac{1}{2} \left(x^{\frac{3}{2}} \right) \Big|_1^9 \\ &= \frac{1}{2} \left(9^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{1}{2} (27 - 1) = \frac{26}{2} = 13. \end{aligned}$$

- (2) [4] Consider the function $g(x, y) = 2x^2 - y$.
(a) Compute $g(1, 3)$.
(b) Give the equation of the level curve of $g(x, y)$ that contains the point $(1, 3)$.

Solution (a). $g(1, 3) = 2 \cdot 1^2 - 3 = 2 - 3 = -1$.

Solution (b). The equation of the level curve will be $g(x, y) = g(1, 3)$, which is given by

$$2x^2 - y = -1.$$

Remark. Notice that this can be written as $y = 2x^2 + 1$, which is the equation of a parabola that could have been easily sketched had you been asked to do so.