Quiz 14 Solutions, Math 220, Professor David Levermore
Wednesday, 1 December 2010

(1) [5] Let $g(x,y) = x^3y + 2xy^2$. Find $\dfrac{\partial g}{\partial x}$, $\dfrac{\partial^2 g}{\partial x^2}$, and $\dfrac{\partial^2 g}{\partial x \partial y}$.

Solution. Because $g(x,y) = x^3y + 2xy^2$, you find that

$$\begin{aligned}
\frac{\partial g}{\partial x}(x,y) &= 3x^2y + 2y^2\,,\\
\frac{\partial^2 g}{\partial x^2}(x,y) &= \frac{\partial}{\partial x}\big(3x^2y + 2y^2\big) = 6xy\,,\\
\frac{\partial^2 g}{\partial x \partial y}(x,y) &= \frac{\partial^2 g}{\partial y \partial x}(x,y) = \frac{\partial}{\partial y}\big(3x^2y + 2y^2\big) = 3x^2 + 4y\,.
\end{aligned}$$

(2) [5] Find all points (x,y) where the function $f(x,y) = \frac{1}{3}x^3 - 3y^2 - 9x + 6y$ has a possible relative maximum or relative minimum. (You do not have to determine if these are relative maximums or relative minimums.)

Solution. Because $f(x,y) = \frac{1}{3}x^3 - 3y^2 - 9x + 6y$, you find that

$$\frac{\partial f}{\partial x}(x,y) = x^2 - 9\,, \qquad \frac{\partial f}{\partial y}(x,y) = -6y + 6\,.$$

The points where $f(x,y)$ has a possible relative maximum or relative minimum are found by setting these partial derivatives to zero, i.e. by setting

$$0 = x^2 - 9 = (x-3)(x+3)\,, \qquad 0 = -6y + 6 = -6(y-1)\,.$$

We find that $x = \pm 3$ and $y = 1$. The points where $f(x,y)$ has a possible relative maximum or relative minimum are therefore $(-3,1)$ and $(3,1)$.

Remark. If you apply the Second Derivative Test found on page 367 of the book, you would find that $f(x,y)$ has a relative maximum at $(-3,1)$ while $f(x,y)$ has neither a relative maximum nor a relative minimum at $(3,1)$. However, you did not have to do this.