Quiz 2 Solutions, Math 246, Professor David Levermore
Tuesday, 8 February 2011

(1) [5] Find the solution of the initial-value problem

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 5t^4 e^{-x}\,, \qquad x(0) = x_I \quad \text{for some } x_I \text{ in } (-\infty, \infty)\,.$$

Give its interval of definition (existence) in terms of x_I.

Solution. This equation is separable. Its separated differential form is $e^x\,\mathrm{d}x = 5t^4\,\mathrm{d}t$, whereby

$$\int e^x\,\mathrm{d}x = \int 5t^4\,dt\,.$$

By integrating both sides we find that $e^x = t^5 + c$. The initial condition then implies that $e^{x_I} = 0^5 + c$, whereby $c = e^{x_I}$. The solution is thereby given implicity by

$$e^x = t^5 + e^{x_I}\,.$$

Whenever $t^5 + e^{x_I} > 0$ this can be solved explicitly for x to obtain

$$x = \log\bigl(t^5 + e^{x_I}\bigr)\,.$$

The condition $t^5 + e^{x_I} > 0$ implies that $t^5 > -e^{x_I}$, which implies that $t > -e^{\frac{1}{5}x_I}$. Therefore the interval of definition is $(-e^{\frac{1}{5}x_I}, \infty)$.

(2) [5] Sketch the phase-line portrait for the equation

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -z(z-2)^2(z-4)\,.$$

Identify every stationary (equilibrium) point as either stable, unstable, or semistable. (You do not have to find the solution!)

Solution. The stationary points are found by setting

$$0 = -z(z-2)^2(z-4)\,.$$

The stationary points are therefore $z = 0$, $z = 2$, and $z = 4$. The phase-line portrait is

