APROXIMATING THE DYNAMICS OF A PERIODICALLY DRIVEN DUFFING OSCILLATOR WITH WHITE NOISE BY A MARKOV CHAIN

Daniel Yuan*, Maria K. Cameron**

*Montgomery Blair High School **University of Maryland, College Park

INTRODUCTION

Energy harvesters and rotors follow a certain time-dependent differential equation. The time-dependent part of the equation comes from the rotation of the systems, and is represented by a certain periodic forcing in the equation.

This equation contains two attractor states (states that map to the same state after one period of forcing): a high-amplitude attractor state and a low-amplitude attractor state. Energy harvesters prefer to be in the high-amplitude attractor, whereas rotors prefer to be in the low-amplitude attractor. However, the system occasionally flips between attractors due to disturbances from the environment.

Markov Chains is a technique that can consider a finite number of states, and can predict the probability that a certain state go to any other state after a certain time interval.

Transition-path theory is a theoretical framework for describing rare events in complex systems. It allows us to analyze how often a system moves from certain state to another, such as a low-amplitude attractor to a high-amplitude attractor.

This study analyzes a certain differential equation with periodic forcing using Markov Chains and Transition Path Theory.

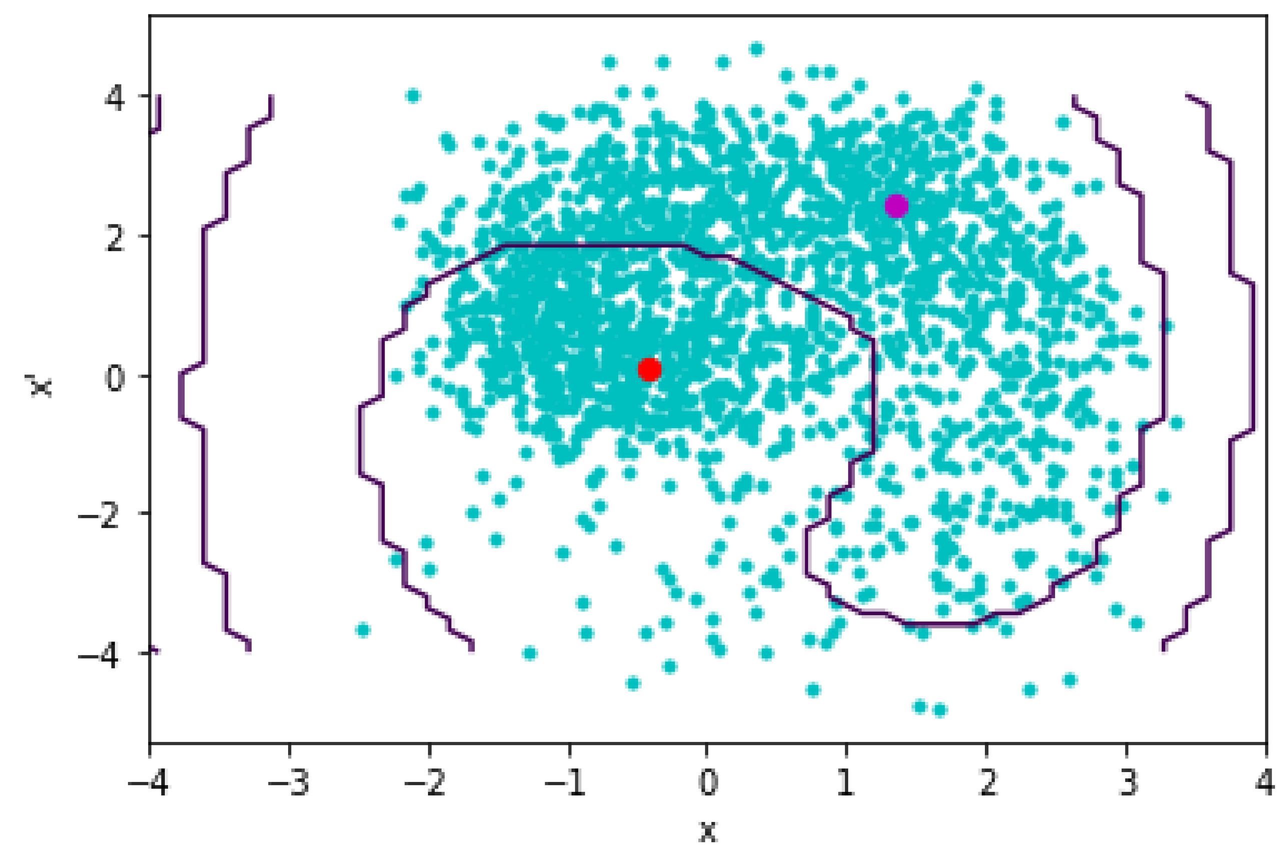

Figure 1: Point Cloud with Attractors and Basin Boundaries

PROCEDURES

The following stochastic differential equation (SDE) was used for analysis:

$$x'' + 0.1x' + x + 0.3x^3 = 0.4\cos(1.4)t + \sqrt{0.01}\eta_t \quad (1)$$

The following SDE was used for creating the point cloud:

$$x'' + 0.1x' + x + 0.3x^3 = 0.4\cos(1.4)t + \sqrt{0.05}\eta_t \quad (2)$$

We create a point cloud with a Stochastic Matrix as follows:

1. Find the 2 attractors of the differential equation (states where after one period, $T = \frac{2\pi}{1.4}$, ends up at the same state)
2. Using the second equation, run a stochastic trajectory $N_{pr} = 1000$ periods long. Record the state at the end of each period. The resulting $2N_{pr}$ recorded states, along with the 2 attractors, by plotting each state as (x, x'), form the point cloud.
3. From each state in the point cloud, using the first equation, run $N_{tr} = 250$ trajectories, each one period long, and from each end state, find the smallest Euclidean distance to a point in the point cloud and consider it to have ended at that point instead.
4. Create a Stochastic Matrix such that

$$P_{ij} = \frac{\text{number of trajectories from state i to state j}}{\text{total number of trajectories from state i}} \quad (3)$$

5. Compute the committor, the backward committor, the probability current and the probability density of reactive trajectories, and the transition rate.

RESULTS

Attractors and Forward Committor

Shown in the figures is the point cloud, the committor, the backwards committor, and the probability density of reactive trajectories.

In Figure 1, the red point is the low-amplitude attractor, and the magenta point is the high-amplitude attractor. The purple lines represent basin boundaries.

The **forward committor function** at a certain state is the probability that the state will reach one of the states in the magenta circle before any state in the cyan circle as shown in Figure 2.

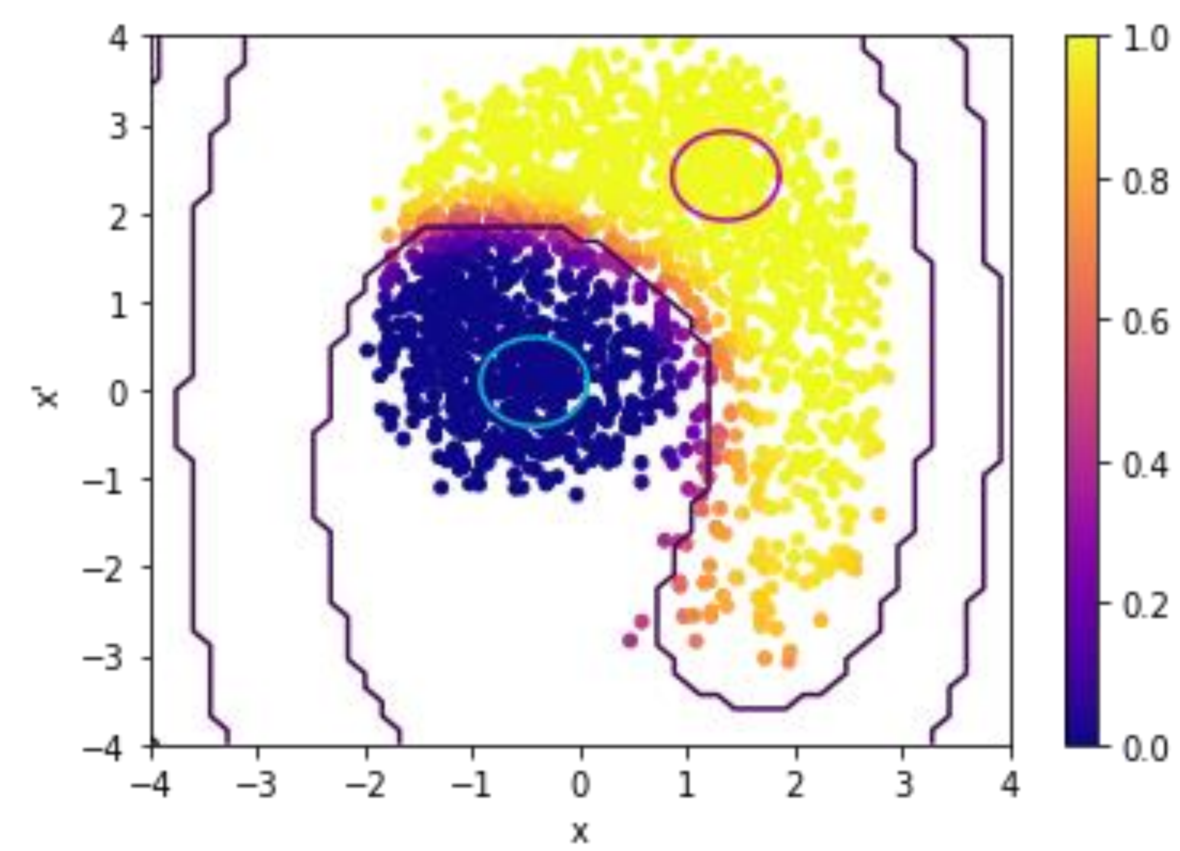

Figure 2: Forward Committor Function

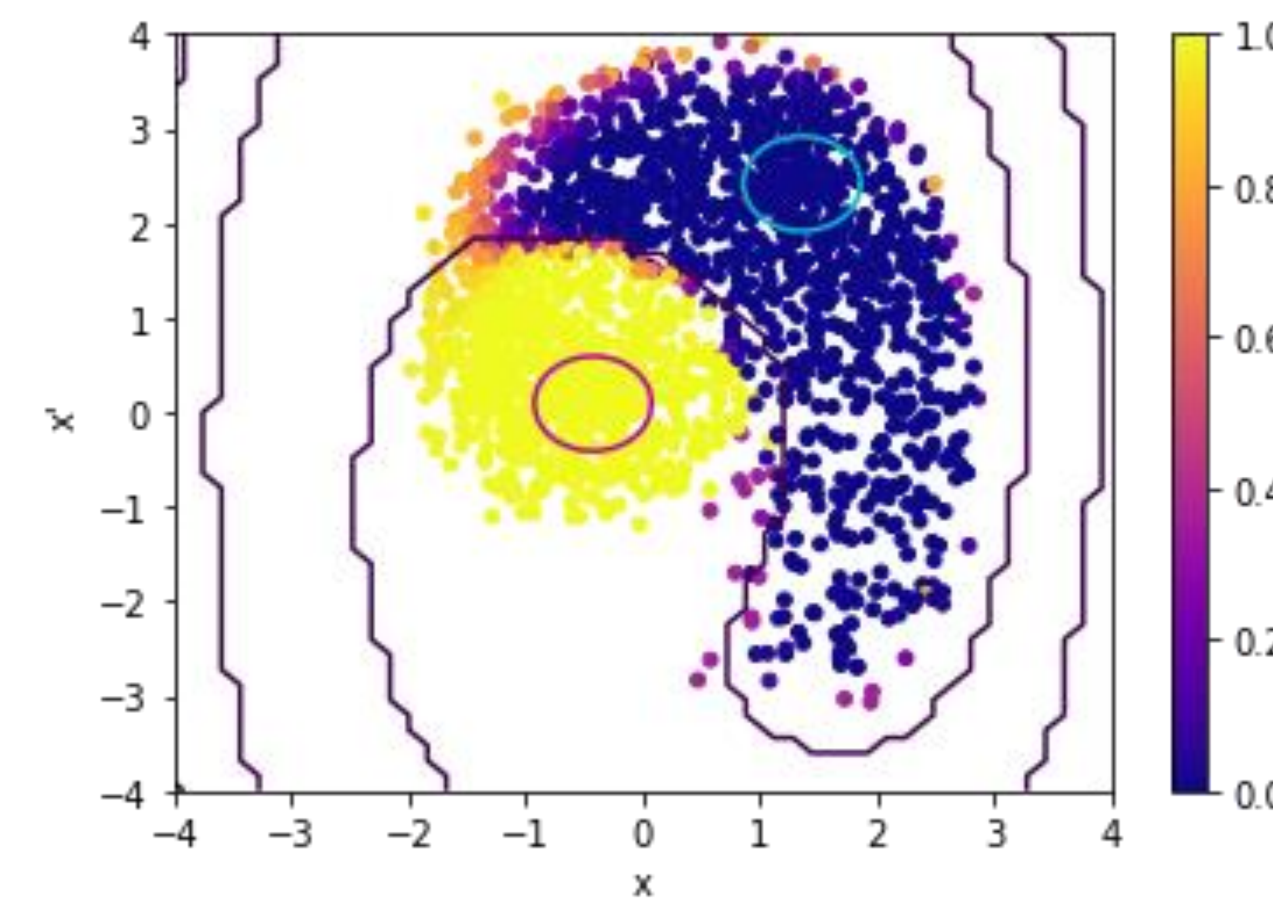

Figure 3: Backward Committor Function

FUTURE RESEARCH

In the future, I will try a different procedure that also uses Markov Chains and Transition Path Theory, but this procedure creates the point cloud as it creates the stochastic matrix. I will then compare its efficiency and accuracy with my first procedure.

RESULTS

Backward Committor and Probability Density of Reactive Trajectories

The **backward committor function** at a certain state is the probability that the state came from one of the states in the magenta circle before any state in the cyan circle as shown in Figure 3.

Shown in Figure 4 is the **probability density of reactive trajectories**, which is, at a given state, the probability that at a certain time step, the system is at that current state and is in a reactive trajectory from the magenta circle to the cyan circle. That is, it came from the magenta circle and is going to the cyan circle.

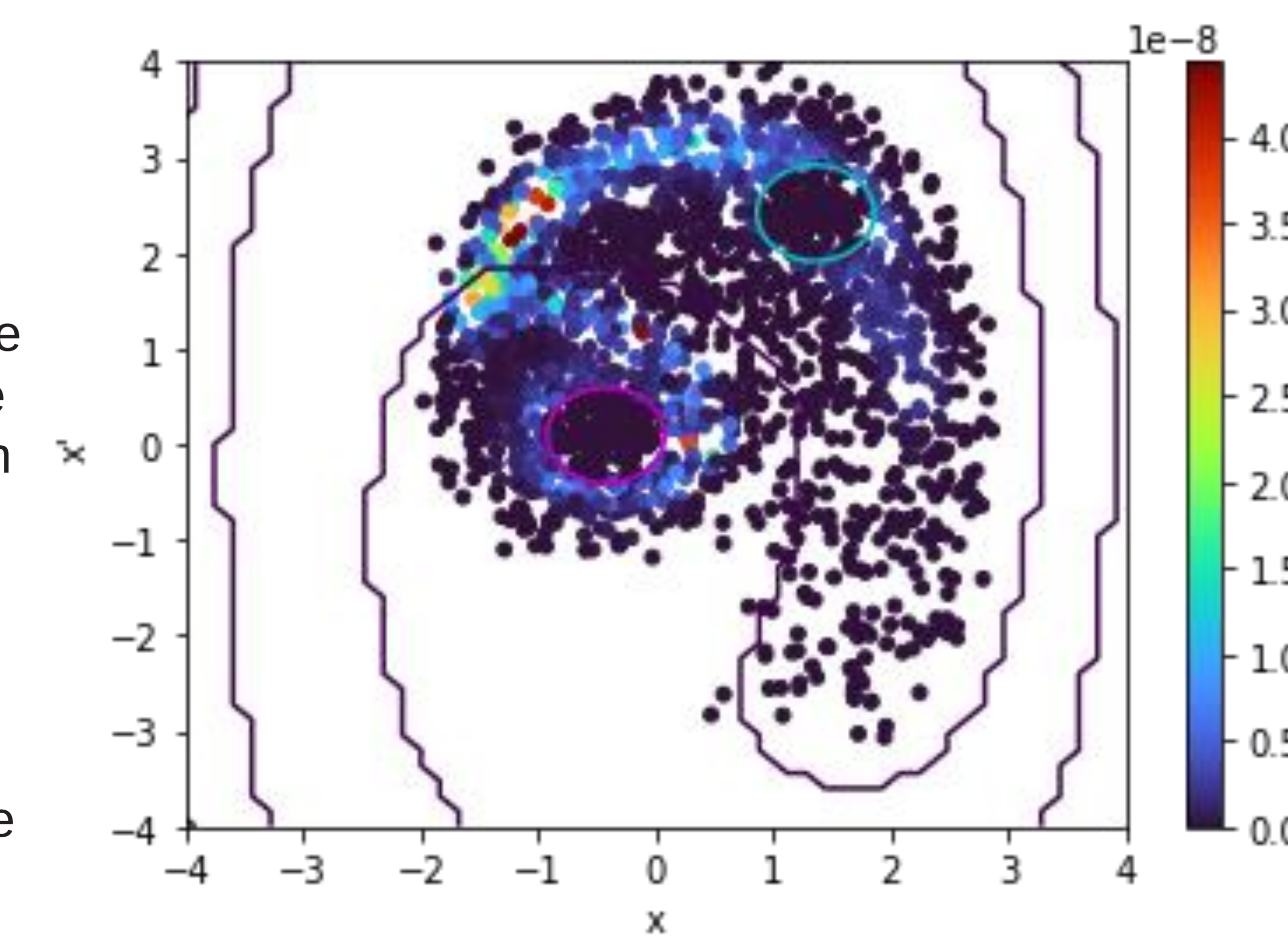

Figure 4: Probability Density of Reactive Trajectories