Geometric Aspects of a Fluid Model for Electrons

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Dyakonov-Shur Model: Microstrip Geometry

Background: The original Dyakonov-Shur paper from 1993 described the fluid-like behavior of electron flow in microdevices. With current technological trends causing devices to become thinner and smaller, there is interest in understanding how electrons behave in 2-dimensional materials like graphene. We are primarily interested in how **instability may arise in the system.** Instability is interesting because it makes the system **easier to observe**.

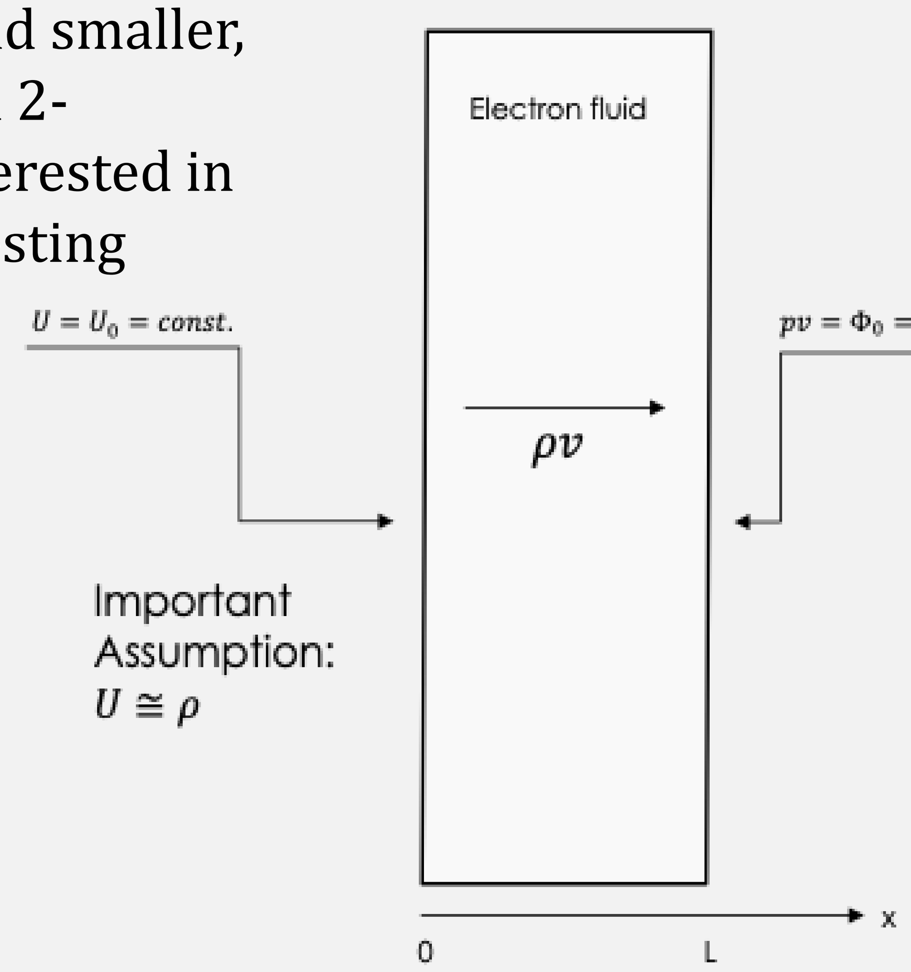

Fig. 1: The geometry and boundary conditions described by Dyakonov, Shur

Model Assumptions:
- $U_1(x) \ll U_s, \Phi_1(x) \ll \Phi_s$
- Ansatz Solutions
  - $U(x,t) = U_1(x)e^{-i\omega t} + U_s$
  - $(Uv)(x,t) = \Phi(x,t) = \Phi_1(x)e^{-i\omega t} + \Phi_s$

Partial Differential Equations:
- Euler (Momentum) Equation: $\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{e}{m}\frac{\partial \rho}{\partial x}$
- Continuity Equation: $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0$

Alpha Parameter Boundary Condition: In the original model, we find instability in the subsonic regime ($v_0 < s$) and stability in the supersonic regime ($v_0 > s$). This behavior is swapped when the constant boundary conditions are swapped. We introduce the parameter α to examine how the instability criteria changes when we mix the two boundary conditions. This is accomplished by taking a convex combination of the flux and electrostatic potential at both boundaries:
- $(1-\alpha)v_0 U(0,t) + \alpha\Phi(0,t) = \Phi_0$
- $\alpha v_0 U(L,t) + (1-\alpha)\Phi(L,t) = \Phi_0$

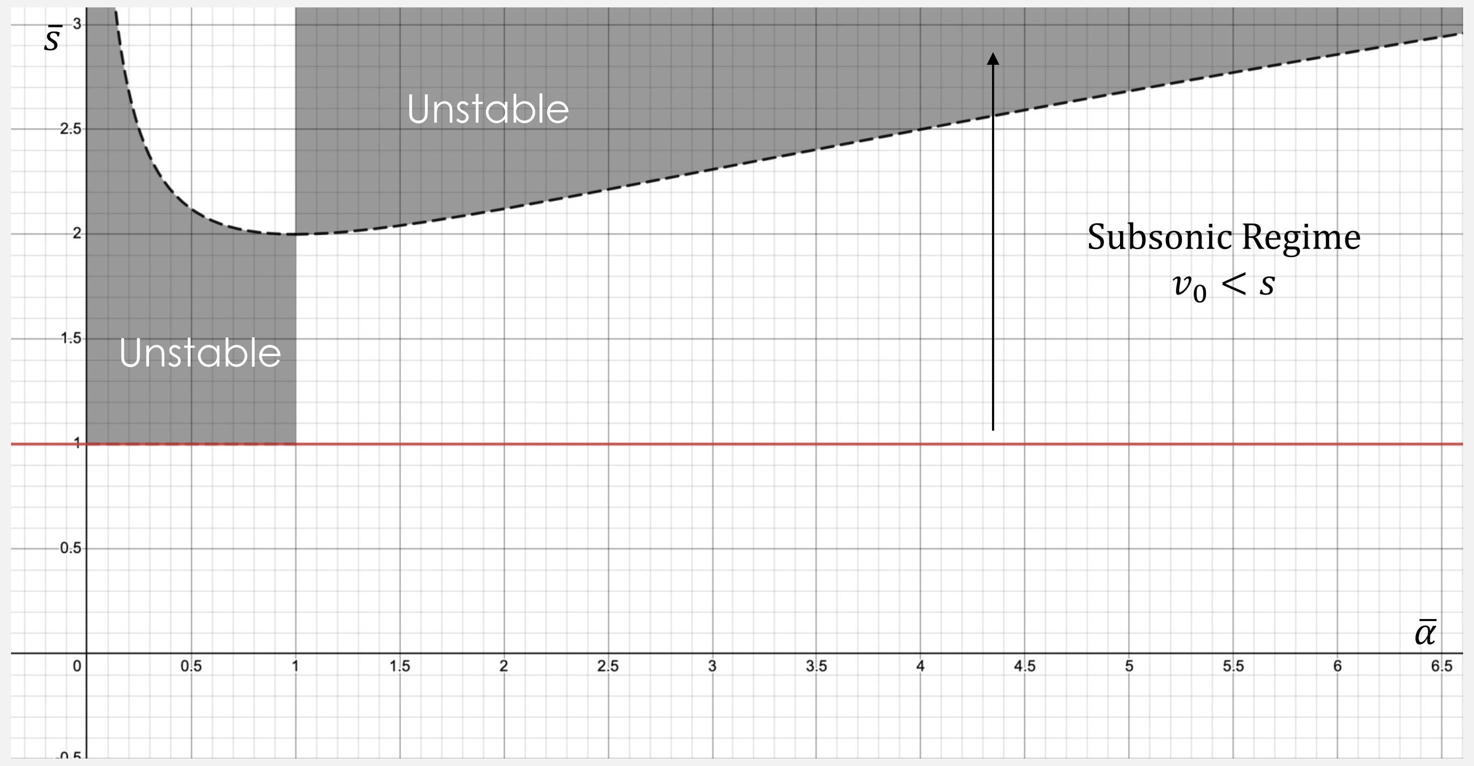

Figure 2: Phase diagram showing regions of stability for the system

Robin Conditions: The second set of new boundary conditions we imposed on the system were Robin Conditions, which involved a new "impedance" term applied **only** to the **left boundary** (e.g. the left boundary condition became $U(x,t) + Z\frac{\partial U(x,t)}{\partial x} = U_0$). By perturbation in Z, we found that this condition yielded the following value for ω:

$$\omega = \frac{s^2 - v_0^2}{2s(L-Z)}\pi n + i\frac{s^2 - v_0^2}{2s(L-Z)}\ln\left|\frac{s+v_0}{s-v_0}\right|.$$

which tells us that the introduction of the Robin Condition had the same effect on the instability criterion of the system as a renormalization of the length scale for the microstrip.

The Annulus Problem: Effect of Curvature

Background: The microstrip electron flow model focused on a geometry without curvature. Next, we introduced curvature by looking at an annulus. This systems maintains the 2 boundary conditions and a one-dimensional motion for the flow. We want to find how the **instability or stability criterion** for this system.

Model Assumptions:
- Electron flow is rotationally symmetric
- $\Psi(r,t) = \rho v_r(r,t)$
- $U(r,t) \cong C\rho(r,t)$
- **Ansatz Solution:**
  - $\Psi(r,t) = \Psi_1(r)e^{-i\omega t} + \Psi_s(r), \ \Psi_1 \ll \Psi_s$
  - $U(r,t) = U_1(r)e^{-i\omega t} + U_s(r), \quad U_1 \ll U_s$

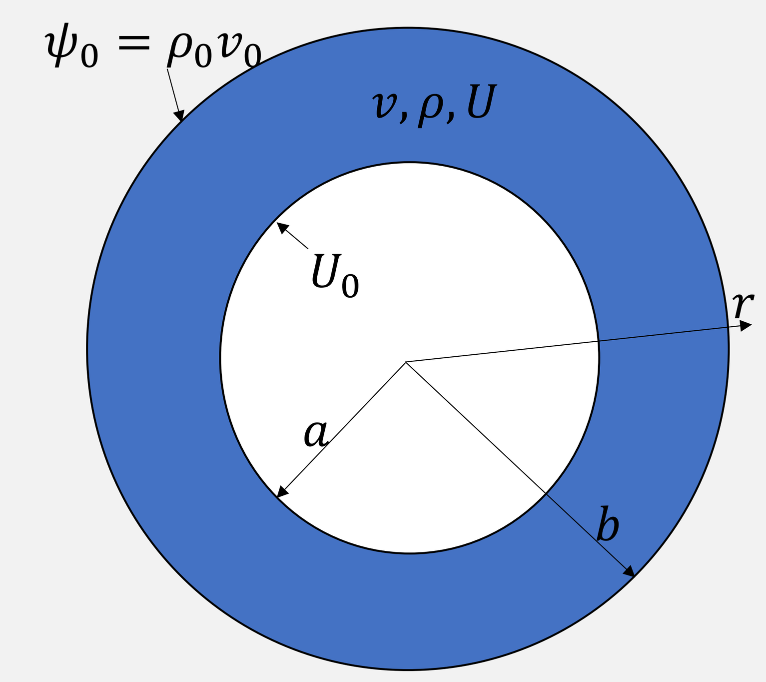

Figure 1: Geometry of the Annulus with the boundary conditions.

Equations of Motion for Radial Coordinate System:

Euler Equation: $\frac{\partial v_r}{\partial t} + \frac{1}{2}\frac{\partial v_r^2}{\partial r} = -\frac{e}{m}\frac{\partial U}{\partial r}$

Continuity Equation: $\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) = -\frac{e}{m}\frac{\partial U}{\partial r}$

Boundary Conditions:

$U(a,t) = U_0 \qquad \Psi(b,t) = \Psi_0$

Steady State Solutions of the System: The steady state equations can be represented as the following:
- Electrostatic Potential:
- $U_s(r) = \frac{U_0\lambda}{3}\left[2\cos\left(\frac{\theta(r)}{3}\right) + 1\right]$,
  $\theta(r) = 1 - \frac{27}{2}\cdot\frac{\lambda-1}{\lambda^3}\left(\frac{a}{r}\right)^2$
- Velocity Flux:
- $\Psi_s(r) = \frac{b}{r}\psi_0$

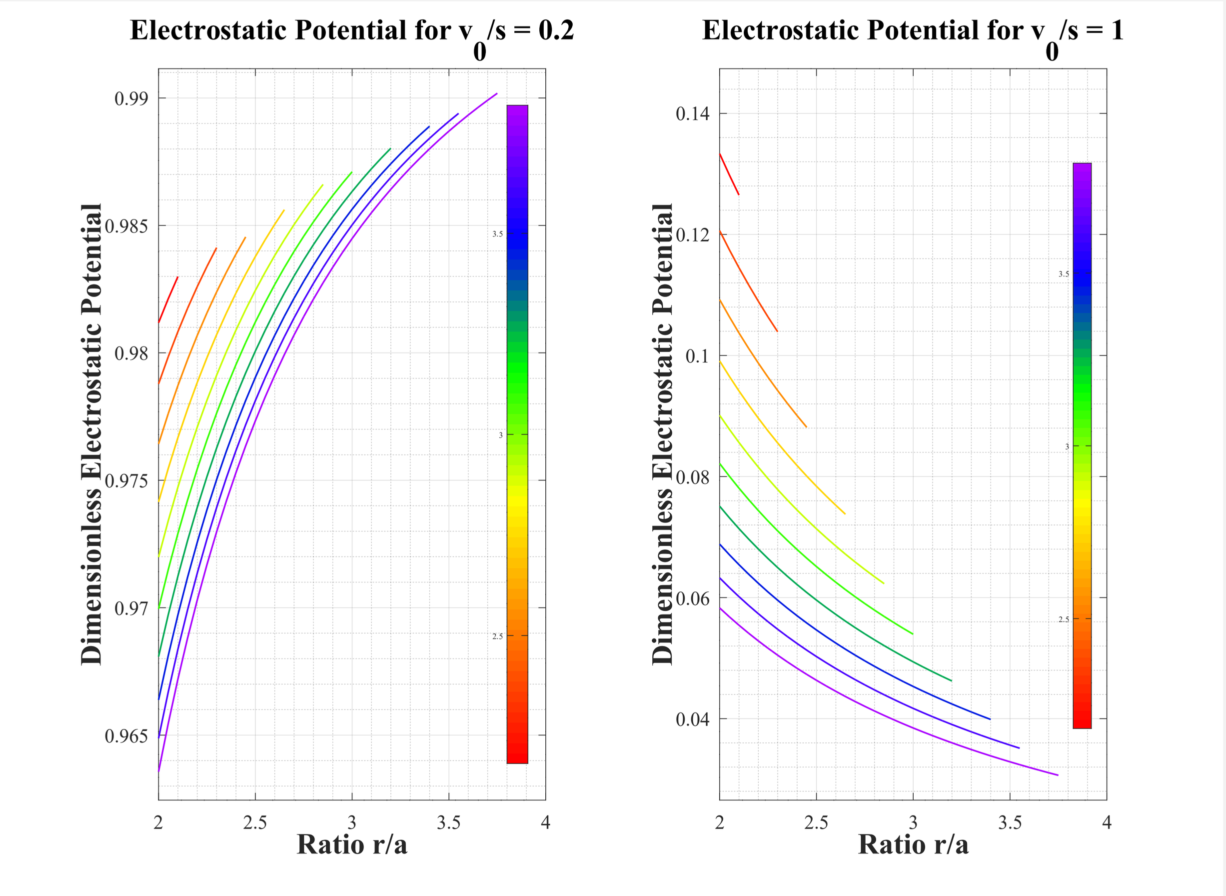

Figure 2: Electrostatic Potential for different values of $\frac{b}{a}$.

Transient State Formulation of the System: The partial differential equations were linearized around $e^{-i\omega t}$ of the Ansatz solution. Then, the equations where **decoupled** and simplified to **canonical form**, where $\Psi_1 \mapsto Y$.

- $Y''(x) + V(\varepsilon x)Y(x) = 0$
- $V(\varepsilon x) = \frac{\omega^2(s^2+2\varepsilon v_0^2)}{(s^2-(1+2\varepsilon)v_0^2)^2} + \left\{\frac{L\omega^2 v_0}{(s^2-(1+2\varepsilon)v_0^2)^2} - i\omega v_0^5\left[\frac{\left(\frac{s}{v_0}\right)^4 + \left(\frac{s}{v_0}\right)^2 + 3 - 3\left(\frac{v_0}{s}\right)^2 + 2\left(\frac{v_0}{s}\right)^4}{(s^2-(1+2\varepsilon)v_0^2)^3}\right]\right\}\frac{\varepsilon}{L} + \omega^2 v_0^2\left[\frac{-1+\left(\frac{v_0}{s}\right)^2 + 2\left(\frac{v_0}{s}\right)^4}{(s^2-(1+2\epsilon)v_0^2)^2}\right]\frac{\varepsilon}{L}x$

Conclusion: The annulus problem shows the steady state terms are functions of the radius. Applying the limit for the unsteady state as $\varepsilon \to 0$ returns the original model. Finally, after solving the system with either the WKB Method or Aireys functions, we expect the instability or stability to depend on the curvature ε of the geometry normalizing the initial velocity of the flow.

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