1. Consider the three vectors $\bar{u} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, $\bar{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\bar{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

   (a) Show that $\bar{u}$, $\bar{v}$ and $\bar{w}$ are not linearly independent using the strict definition.

   (b) Explain why any pair of these vectors is linearly independent.

   (c) Give an example of a fourth vector $\bar{x}$ which, when paired with any of $\bar{u}$, $\bar{v}$ or $\bar{w}$, yields a pair which is linearly dependent.

2. Define

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -4 & 1 \\ 1 & 2 & -1 \end{bmatrix}$$

   (a) Solve $A\bar{x} = \bar{0}$. Express your answer in parametric form.

   (b) It is a fact that $\begin{bmatrix} -5 \\ 0 \\ 2 \end{bmatrix}$ is a solution to $A\bar{x} = \begin{bmatrix} -5 \\ 12 \\ -7 \end{bmatrix}$. Find all solutions to this equation. Express your answer in parametric form.

   (c) Find some $\bar{y}$ so that $\bar{y}$ is not in the span of the columns of A and justify why it is not.

3. (a) Consider the transformation determined by the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ -1 & 2 & 1 & 0 \end{bmatrix}$$

   Rewrite this transformation in the form $T(x_1, ...) = (...)$ with as many x_i as necessary for the dimensions.

   (b) Suppose a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is given by first compressing by a factor of $\frac{1}{2}$ in the x_1-direction and then rotating 180° about the origin. Find the matrix for T and use this to find $T(-2, 3)$.

4. (a) Show that the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $T(x_1, x_2) = (2x_1 - x_2, x_2, -3x_1)$ is a linear transformation.

   (b) Suppose a linear transformation T corresponds to a 4×5 matrix. Explain why T is not one-to-one.

5. (a) Use the inverse of a matrix to solve the vector equation

$$x_1 \begin{bmatrix} 1.5 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -0.5 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

   (b) Determine all values of h for which the following matrix is invertible.

$$\begin{bmatrix} -1 & 1 & 0 \\ 2 & 0 & 4 \\ 0 & 3 & h \end{bmatrix}$$