Solution to the review problems
Final Exam

1. (a) Area= $\frac{\sqrt{22}}{2}$

   (b) $\mathbf{L} = (1, 2\sqrt{2}, 2)$ is in the direction of ℓ. $\mathbf{N} = (2\sqrt{2}, -3, 2\sqrt{2})$ is normal to P. We check that $\mathbf{L} \cdot \mathbf{N} = 0$.

   (c) Distance $= 1$.

2. (a) $(5/2, -1, 0)$

   (b) $(2, 0, 1)$

   (c) $\sqrt{5}$

3. (a) 20π

   (b) $a_T = 0$, $a_N = 10$.

4. $\mathbf{v}(t) = -\sin t\mathbf{i} + \cos t\mathbf{j} - (-\sin t + \cos t)\mathbf{k}$, $\mathbf{a}(t) = -\cos t\mathbf{i} - \sin t\mathbf{j} - (-\cos t - \sin t)\mathbf{k}$. $\mathbf{r} \times \mathbf{v} = \mathbf{i} - \mathbf{j} + \mathbf{k}$, so $\frac{d}{dt}(\mathbf{r} \times \mathbf{v}) = \mathbf{0}$.

5. Critical points $(0, 0)$ (saddle) and $(2, 2)$ (relative minimum).

6. (a) $\nabla f = ye^z\mathbf{i} + xe^z\mathbf{j} + xye^z\mathbf{k}$

   (b) $D_u f(1, 1, 0) = \frac{2}{\sqrt{6}}$

   (c) $(2, 2, -\ln 4)$

7. $-(x - 1) + 2y - 2(z + 1) = 0$

8. $\iint_R dx\,dy = \int_0^8 \int_{v/3}^{v/2} \frac{1}{3}u^{-2/3}\,du\,dv = 12\left(\frac{1}{2^{1/3}} - \frac{1}{3^{1/3}}\right)$.

9. $= \int_0^9 \int_0^{\sqrt{x}} y\cos(x^2)\,dy\,dx = \frac{1}{4}\sin 81$

10. $\iint_S \mathbf{F} \cdot \mathbf{n}\,dS = \frac{243\pi}{2}$

11. (a) $\mathbf{curl}\,F = \mathbf{0}$.

    (b) $\mathbf{F} = \nabla f$ with $f(x, y, z) = e^x \cos z + \frac{y^2}{2}$.
    $\int_C \mathbf{F} \cdot d\mathbf{r} = f(1, 1, 1) - f(1, 0, 0) = e(\cos 1 - 1) + \frac{1}{2}$.

12. $\iint_\Sigma \mathbf{F} \cdot \mathbf{n}\,dS = \int_0^1 \int_0^1 \int_0^x e^x + 2y + e^z\,dz\,dy\,dx = e - 1/2$

13. $= \iint_R 3 - 2y - 2\,dA = 9\pi$

14. (a) $I = \int_0^{2\pi} \int_0^{\pi/3} \int_2^2 \rho^2 \cdot \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta = \frac{31\pi}{5}$.

    (b) $\iint_\Sigma \mathbf{F} \cdot \mathbf{n}\,dS = \iiint_D \mathrm{div}\mathbf{F}\,dV = 3I = \frac{93\pi}{5}$