Fall 2012 - Math 241

MATLAB Assignment #2 - Due Oct. 18

Differential calculus with Matlab. The `diff` command can be used to compute derivatives. You must always remember to declare the variable (t in the example below) as a symbolic variable first (we do that with the command `sym` below. You can also use `syms` as in the previous assignment).

```
>> t=sym('t','real')
>> f=t*sin(3*t^2)
>> diff(f)
```

You can also differentiate vector-valued functions. For instance, if

$$\vec{r}(t) = t\vec{i} + t\cos(t^2)\vec{k},$$

then you can compute $\vec{v}(t) = \vec{r}\,'(t)$ as follows:

```
>> r=[t, 0, t*cos(t^2)]
>> v=diff(r)
```

To compute the value $\vec{v}(2)$, you can use `subs(v,2)`. The speed of the object at time t is given by the norm of $\vec{v}$. However, the command `norm` does not work with symbolic variable. So to compute $||\vec{v}(t)||$, we use `sqrt(dot(v,v))`.

Integral calculus with Matlab. Integrals are computed with the command `int`. For instance, `int(r)` computes an indefinite integral of $\vec{r}$. The command `int(r,0,1)` computes the definite integral over the interval $[0, 1]$.

Note that `int` attempts to find an exact formula. If that fails (and it will fail for complicated functions), you can use the command `double` to get a numerical result. Try to the following example:

```
>> t=sym('t','real')
>> g=exp(cos(t))
>> int(g,0,1)
>> double(int(g,0,1))
```

You can also compute the integral of vector valued functions. For instance, here is how to compute

$$\int_0^1 e^t\vec{i} + t^2\vec{j} + \cos(t)\vec{k}\,dt$$

```
>> F=[exp(t),t^2,cos(t)]
>> int(F,0,1)
```

Plotting surfaces. The graph $z = f(x, y)$ can be drawn with `ezmesh`. For instance, to draw the surface

$$z = e^x - y\sin(2x), \quad -5 \le x \le 2, \quad -2 \le y \le 2$$

you would type

```
>> ezmesh('exp(x)-y*sin(2*x)',[-5 2 -2 2])
```

You can increase the resolution of the picture by increasing the number of grid points. For instance, try

```
>> ezmesh('exp(x)-y*sin(2*x)',[-5 2 -2 2],200)
```

Level sets. You can also plot some level sets of a function $f(x, y)$ with the command `ezcontour`. Try for instance

```
>> ezcontour('x^2-y^2')
```

and make sure you understand the picture.

Problem 1

(a) Enter the vector-valued function $\vec{r}(t) = ((\frac{t}{\pi}) - \sin(t), 1 - \cos(t), \sin(t))$ (use `pi` for π).

(b) Find the unit tangent vector $\vec{T}(\pi)$ to the curve traced out by $\vec{r}(t)$ at $t = \pi$.

(c) Find the unit normal vector $\vec{N}(\pi)$ to the curve at $t = \pi$.

(d) Using `ezplot3` and `quiver3`, as shown in the first assignment, plot on the same graph the curve traced out by $\vec{r}(t)$ for $0 \le t \le 2\pi$, the unit tangent vector and the unit normal vector at the point $\vec{r}(\pi)$.

Problem 2 Let $\vec{r}(t) = (t^2e^t, \sin(2t) - t^6, -\ln(t)\cos(t))$ represent the position of a particle at time $t > 0$.

(a) Find the speed, velocity, and acceleration of the particle at time $t = 1$ (the command for $\ln(t)$ is `log(t)`).

(b) Find the tangential and normal component of the acceleration at time $t = 1$.

Problem 3 Find the length of the curve parametrized by $\gamma(t) = (t\cos(t), t, t\sin(t))$ over the interval $0 \le t \le 4\pi$.

Problem 4 Let $f(x, y) = \cos x \sin y$.

(a) Plot the graph $z = f(x, y)$ over the domain $-5 \le x \le 5$, $-5 \le y \le 5$.

(b) Plot some level curves of the function f over the same domain.

Problem 5 Plot the following quadrics:

(a) $z = 2x^2 - y^2$, for $-1 \le x \le 1$, $-1 \le y \le 1$ (hyperbolic paraboloid).

(b) $z = x^2 + 3y^2$, for $-2 \le x \le 2$, $-1 \le y \le 1$ (elliptic paraboloid).

(c) $z^2 = x^2 + y^2 + 1$, for $-5 \le x \le 5$, $-5 \le y \le 5$ (hyperboloid of two sheets) **Hint**: Plot the graphs of $f(x, y) = \sqrt{x^2 + y^2 + 1}$ and $f(x, y) = -\sqrt{x^2 + y^2 + 1}$ on the same graph, using the `hold on` command.

Remember to use the command `axis equal` to get a better view.