Fall 2012 - Math 241

MATLAB Assignment #3 - Due Nov. 8

Multivariable calculus with Matlab. First we recall that you can differentiate a function of a single variable, such as $f(x) = x^2$, using the `diff` command as follows:

```
>> syms x
>> f=x^2
>> diff(f)
```

For functions of more than one variable, you can differentiate with respect to any of its variables using the same command `diff`. For instance, the following commands set up the function $f(x, y) = x\cos(x + y) - e^{xy}$ and calculate $\frac{\partial f}{\partial x}$:

```
>> syms x y
>> f = x*cos(x+y)-exp(x*y)
>> fx = diff(f,x)
```

The gradient of f is then given by `gradf=[diff(f,x),diff(f,y)]`.

To evaluate the value of a function, we use the command `subs`. For instance, `subs(f,{x y},{0 1})` will evaluate $f(0, 1)$ for the function f above (and `subs(gradf,{x y},{0 1})` will return the value of the gradient of f at the point $(0, 1)$).

The `subs` command may give a symbolic answer. To get an actual number use the command `double` which converts a symbolic number to a number:

```
>> double(subs(f,{x y},{0 1}))
```

Iterated Integrals: We can evaluate double and triple integrals by using iterated single integrals. Matlab will do this by iterating the `int` command. You can compute $\int_0^1 \sin(xy)\,dx$, with the command `int(sin(x*y),x,0,1)` (note that you must specify the variable you are integrating). So to compute an iterated integral, such as $\int_0^1 \int_{x^2}^x x^2y^3\,dy\,dx$, you put an `int` command inside an `int` command:

```
>>syms x y
>>int(int(x^2*y^3,y,x^2,x),x,0,1)
```

and if Matlab cannot find a symbolic result, you might have to add a `double` command.

Extreme Values and Critical points. To find the critical points of a function f, you have to solve $\vec{grad} f = \vec{0}$, which is really a system of equation. You can actually use Matlab to solve a system of equations using the `solve` command. Matlab will try to solve symbolically. If it succeeds, it will find a list of all solutions. But the equations may be too complicated to solve symbolically, in which case it will try to find numerical solutions. In this case it may not find all solutions, so you should be wary. For example

```
>> syms x y
>> f = x - 3/y
>> g = 3*y+x-7
>> [ax ay] = solve(f,g)
```

will solve the equations $f(x, y) = 0$, $g(x, y) = 0$ exactly and find the two symbolic solutions. ax will be a list of the x values and ay will be a list of the y values (so the points `[ax(1),ay(1)]` and `[ax(2),ay(2)]` solve the system of equations $f = 0$, $g = 0$). However if we do

```
>> h = cos(x*y)-2^x+3
>> [ax ay]=solve(g,h)
```

Matlab won't be able to solve the equations $g = 0$, $h = 0$ exactly and will return a numerical solution.

We can now use Matlab to find the critical points of a function. Consider for instance the function $f(x, y) = x^3 - 12x + (x - y)^2 + 7$. The following commands find the critical points of f:

```
>> syms x y
>> f = x^3-12*x+(x-y)^2+7
>> fx = diff(f,x)
>> fy = diff(f,y)
>> [ax ay] = solve(fx,fy)
```

In order to classify the points as relative max, relative min or saddle point, we can define

```
>> D = diff(fx,x) *diff(fy,y) - diff(fx,y)^2
>> T = [ ax ay subs(D, {x,y}, {ax,ay}) subs(diff(fx,x), {x,y}, {ax,ay}) ]
>> double(T)
```

The next to last line takes the list of solutions you found in ax and ay and makes a table T with four columns, consisting of x, y, D, and f_{xx}. In some cases the results could be unintelligible, so the last line prints out the table T as a table of numbers. From this you can easily read off the type of each critical point.

Problem 1 (a) Find f_x, f_y, f_{xx}, f_{yy}, and $f_{xy} = f_{yx}$ for the function

$$f(x, y) = x^y \sin(x)$$

(b) Find $f_{xx}(\pi/2, \pi)$.

(c) Find the directional derivative of f at the point $(\pi/2, \pi)$ in the direction of the vector $\vec{u} = [1, 1]$.

Problem 2 Evaluate the following double or triple integrals (approximate with `double` if necessary):

(a) $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 4 - x^2 - y^2 \, dy \, dx$

(b) $\int_{0}^{1} \int_{0}^{2\pi} r \sin(\theta r^2) \, d\theta \, dr$

(c) $\int_{1}^{3} \int_{0}^{\pi} \int_{0}^{2} xz \sin(xy) \, dz \, dy \, dx$

Problem 3 Find the volume of the solid region bounded by the two paraboloids $z = 2x^2 + y^2$ and $z = 16 - 2x^2 - y^2$. Set up the problem using rectangular coordinates and find the integral using MATLAB. You do not need to justify how you set up the problem.

Problem 4 Consider the function $f(x, y) = 2 - 2x^2 - y^2$ and let S be the graph of f.

(a) Find the equation of the tangent plane to S at $(0, 1, 1)$

(b) On the same graph, plot S and the tangent plane from (a).

Problem 5 Use Matlab to find and classify all critical points of the function

$$f(x, y) = x - 2y + y^3 - 15y^5 - (x - 2y)^3$$