Fall 2012 - Math 241

MATLAB Assignment #4 - Due Nov. 29

Parametrized Surfaces The command `ezmesh`, which we used in the previous assignments, can also be used to draw parametrized surfaces. For instance, the sphere of radius 1, parametrized (using cylindrical coordinates) by

$$F(s,t) = \sqrt{1-s^2}\sin t\vec{i} + \sqrt{1-s^2}\cos t\vec{j} + s\vec{k}, \qquad 0 \le t \le 2\pi, \quad -1 \le s \le 1,$$

can be drawn with the following command:

```
>> syms s t
>> ezmesh(sqrt(1-s^2)*sin(t),sqrt(1-s^2)*cos(t),s,[-1 1 0 2*pi])
```

(try to plot the same sphere using spherical coordinates).

Plotting vector fields using quiver and quiver3. Matlab can plot vector fields. Suppose we wish to plot $\vec{F}(x,y) = \frac{y}{5}\vec{i} - \frac{x}{5}\vec{j}$. You can plot the vectors corresponding to each point (x,y) with integer coordinates in the range $-6 \le x \le 6$, $-4 \le y \le 7$ as follows:

```
>> [x,y]=meshgrid(-6:1:6, -4:1:7);
>> quiver(x, y, y/5, -x/5)
```

The first line creates the grid of points where we want to draw the vectors. Note that the 1 in between the lower and upper limits of x and y is the increment Δx or Δy that tells Matlab the increments in the coordinate ranges (note also the use of the semicolon which suppresses the output. Do not forget it or your printout will be very long). The last line plots the vector field.

Common error: When executing the command `quiver` above, Matlab processes `x` as an array (or a matrix). This is important to know because some operations have a different syntax in that case: you may need to put a period before some symbol. For instance, if you want to multiply the coefficients of two arrays `x` and `y`, you should write `x.*y` instead of `x*y`. So if $\vec{F}(x,y) = x^2\vec{i} + 2xy\vec{j}$, then

```
>> quiver(x, y, x^2, 2*x*y)
```

will not work, but

```
>> quiver(x, y, x.^2, 2*x.*y)
```

will. If you do not do that, you will likely get an error that says "matrix must be square". Note that you do not need the period for trig, exp and log functions (e.g. `cos(x)` will work).

To plot a 3-dimensional vector field, use a 3-dimensional meshgrid and use the `quiver3` command. For example, $\vec{F}(x,y,z) = y\vec{i} - 2x\vec{j} + z\vec{k}$ is plotted as follows:

```
>> [x,y,z]=meshgrid(-6:1:6, -4:1:7, 0:2:6);
>> quiver3(x, y, z, y, -2*x, z, 0.5)
```

(the 0.5 scales the vectors so that the picture looks nicer.)

Problem 1 (a) Find a parametrization of the hyperboloid of one sheet given by

$$z^2 = x^2 + y^2 - 1, \qquad -2 \le z \le 2$$

(use cylindrical coordinates). Use `ezmesh` to plot this surface.

(b) Let S be the surface of revolution obtained by revolving about the x axis the graph of $y = \cos x$ for $-\pi/2 \le x \le \pi/2$. Find a parametrization for S and plot this surface using `ezmesh`.

Problem 2 A torus is parametrized by

$$\vec{r}(x, y) = ((2 + \cos(y))\cos(x), (2 + \cos(y))\sin(x)), \sin(y))$$

where $0 \le x \le 2\pi$ and $0 \le y \le 2\pi$.

(a) Plot this surface using `ezmesh` (use `axis equal` to get a nicer picture).

(b) Find the surface area of this torus

Problem 3 Plot the vector field

$$\vec{F}(x, y) = \left(\frac{x}{x^2 + y^2}, \frac{1}{x^2 + y^2}\right)$$

on the rectangle $-10 \le x \le 10$, $-5 \le y \le 5$.

Problem 4 Use Matlab to evaluate the line integral

$$\int_C \vec{F} \cdot d\vec{r}$$

where $\vec{F}(x, y, z) = e^y\vec{i} - x^2y\vec{j} + xz^3\vec{k}$ and C is the curve parametrized by

$$\vec{r}(t) = \cos t\,\vec{i} - 2t\,\vec{j} + \sin(3t)\,\vec{k}, \quad 0 \le t \le \pi.$$

Problem 5 Consider the region R bounded by the curve

$$\vec{r}(t) = \sin(2t)\vec{i} + \sin(t)\vec{j}, \quad 0 \le t \le \pi.$$

(a) Use the `ezplot` command to plot this curve and identify the region R.

(b) Use Green's theorem to find the area of the region R (Hint: The vector field $\vec{F}(x, y) = x\vec{j}$ may be helpful)