Math 241 **Midterm #1** **Fall 2012**

Fri. Sept. 28

Instructions: Number the answer sheets from 1 to 5. Fill out **all** the information at the top of **each** sheet (write and sign the Honor Pledge on page 1 only). Answer **one** question on **each** sheet in the correct order: Problem 1 on sheet 1, problem 2 on sheet 2... (Do **not** answer one question on more than one sheet. Use the back of the correct sheet if you need more space).

None of the following are allowed: lecture notes, book, electronic devices of any kind (including calculators, cell phones, etc.)

You may keep with you **one sheet of notes**.

1. Consider the points $A = (1, 3, 2)$ and $B = (3, 0, -1)$.
   (a) [10 pts] Find parametric equations for the line ℓ containing the points A and B.
   (b) [5 pts] Find the intersection of the line ℓ with the plane $z = 0$.

2. (a) [15 pts] Find the equation for the plane $\mathcal{P}$ that contains the point $P_0 = (-4, -1, 1)$ and the line with symmetric equations
   $$\frac{x+1}{4} = \frac{y}{2} = z - 2.$$
   Write your answer in the form $ax + by + cz = d$.
   (b) [10 pts] Find the distance between the point $P = (1, 3, 2)$ and the plane $\mathcal{P}$ defined in part (a).

3. [15 pts] Compute the length of the curve parametrized by
   $$\mathbf{r}(t) = \sin(3t)\mathbf{i} + \cos(3t)\mathbf{j} + 2t^{3/2}\mathbf{k}, \qquad 0 \le t \le 2.$$

4. Consider an object moving with velocity
   $$\mathbf{v}(t) = 2t\mathbf{i} + 2\mathbf{j} + t^2\mathbf{k}.$$
   (a) [5 pts] Find the position $\mathbf{r}(t)$ of this object at time t, assuming that $\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$.
   (b) [15 pts] Find the acceleration $\mathbf{a}(t)$ as well as the tangential and normal components a_T and a_N of the acceleration.
   (c) [10] Find the curvature $\kappa(t)$ of the trajectory of the object at time t.

5. [15 pts] Consider the curve parametrized by
   $$\mathbf{r}(t) = (\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j}.$$
   Find the unit tangent vector $\mathbf{T}(t)$ and the unit normal vector $\mathbf{N}(t)$.