Review problems for the first midterm

These are typical problems that could be on the exam, though there are more problems here than could possibly be on the exam (you can expect 4 or 5 problems on the exam).

The exam may include material that is not on the review and the wording of questions may be somewhat different. Make sure that you do all suggested homework problems and review all the material discussed in class.

1. Consider the vectors
$$\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}, \quad \mathbf{c} = -5\mathbf{j} + \mathbf{k}$$
   (a) Find a **unit** vector perpendicular to $\mathbf{a}$ and $\mathbf{b}$.
   (b) Show that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ all lie on the same plane.
   (c) Show that $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.
   (d) Write $\mathbf{c} = \mathbf{u} + \mathbf{v}$ where $\mathbf{u}$ is parallel to $\mathbf{a}$ and $\mathbf{v}$ is parallel to $\mathbf{b}$.
2. Let l be the intersection of the planes $2x - 3y + 4z = 2$ and $x - z = 1$.
   (a) Find the parametric equations for l.
   (b) Find the symmetric equations for l.
   (c) Find the distance from the origin O to l.
3. Find an equation for the line that is perpendicular to the plane with equation
$$2x - 5y + z = 2$$
and that contains the point $P_0 = (1, -2, 3)$.
4. Find the point on the plane $2x - 4y + z = 10$ which is closest to the point $(1, 3, 0)$.
5. (a) Explain why the lines with symmetric equations
$$\frac{x-2}{2} = \frac{y+3}{3} = z - 5$$
and
$$\frac{x}{2} = \frac{y-2}{3} = z + 1$$
are parallel.
   (b) Find an equation for the plane that contains both lines.
6. A particle has the position function
$$\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + \frac{3}{5}t^{5/3}\mathbf{k}, \quad t > 0.$$
   (a) Find the velocity, speed and acceleration of this particle.
   (b) Determine the tangential and normal components a_T and a_N of the acceleration $\mathbf{a}$ from part (a).
7. Find the position, velocity and speed of an object having acceleration $\mathbf{a}(t) = -32\mathbf{k}$, initial velocity $\mathbf{v}_0 = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and initial position $\mathbf{r}_0 = \mathbf{0}$.
8. Consider the curve C parametrized by the vector
$$\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + \ln t\mathbf{k}, \qquad 1 \le t \le 2.$$
   (a) Find ds/dt.
   (b) Find the length L of C.
9. Find the curvature and radius of curvature of the curve
$$\mathbf{r}(t) = t\mathbf{i} + 2t^{1/2}\mathbf{k}$$
at the point corresponding to $t_0 = 1$.