Solution to the review problems

1. (a) $\mathbf{N} = \frac{\mathbf{a}\times\mathbf{b}}{||\mathbf{a}\times\mathbf{b}||} = \frac{1}{\sqrt{42}}(-4,-1,5)$
   (b) $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} = 0$, so they all lie in the same plane.
   (c) $\mathbf{a}\cdot\mathbf{b} = 0$
   (d) $\mathbf{u} = \mathbf{pr_a c} = \frac{(\mathbf{a}\cdot\mathbf{c})}{||\mathbf{a}||^2}\mathbf{c} = (-2,-2,2)$, $\mathbf{v} = \mathbf{c}-\mathbf{u} = (2,-3,-1) = \mathbf{b}$ (or you can use $\mathbf{v} = \mathbf{pr_b c}$).

2. Let l be the intersection of the planes $2x-3y+4z=2$ and $x-z=1$.
   (a) ℓ has direction $\mathbf{L} = (2,-3,4)\times(1,0,-1) = (3,6,3)$. The point $P_0 = (1,0,0)$ is in the intersection of both planes, so on ℓ. So $x = 1+3t$, $y = 6t$, $z = 3t$.
   (b) $\dfrac{x-1}{3} = \dfrac{y}{6} = \dfrac{z}{3}$
   (c) $d = \dfrac{||\mathbf{L}\times \vec{OP_0}||}{||\mathbf{L}||} = \sqrt{\dfrac{5}{6}}$

3. The line has direction $(2,-5,1)$ so we get
$$\frac{x-1}{2} = \frac{y+2}{-5} = z-3$$

4. $\mathbf{N} = (2,-4,1)$ is normal to the plane, and $P_0 = (5,0,0)$ is on the plane. The point P_1 closest to the point $P = (1,3,0)$ is such that
$$\vec{P_1P} = \mathbf{pr_N}\vec{P_0P} = \left(\frac{-40}{21},\frac{80}{21},\frac{-20}{21}\right)$$
and so $P_1 = \left(\dfrac{61}{21}, -\dfrac{17}{21}, \dfrac{20}{21}\right)$.

5. (a) The lines both have direction vector $\mathbf{L} = (2,3,1)$, so they are parallel.
   (b) $P_0 = (2,-3,5)$ is on the first line and $P_1 = (0,2,-1)$ is on the second. So $\mathbf{L}\times\vec{P_0P_1} = (-23,10,16)$ is normal to the plane containing both lines. Therefore, an equation for the plane is $-23(x-2) + 10(y+3) + 16(z-5) = 0$.

6. A particle has the position function
$$\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + \frac{3}{5}t^{5/3}\mathbf{k}, \quad t>0.$$
   (a) $\mathbf{v}(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + t^{2/3}\mathbf{k}$, speed $= ||\mathbf{v}(t)|| = \sqrt{1+t^{4/3}}$, $\mathbf{a}(t) = -\cos t\,\mathbf{i} - \sin t\,\mathbf{j} + \frac{2}{3}t^{-1/3}\mathbf{k}$
   (b) $a_T = \dfrac{d}{dt}||\mathbf{v}(t)|| = \dfrac{1}{2}\left(1+t^{4/3}\right)^{-1/2}\dfrac{4}{3}t^{1/3} = \dfrac{2t^{1/3}}{3\sqrt{1+t^{4/3}}}$. Since $||a(t)||^2 = 1 + \frac{4}{9t^{2/3}}$, we have $a_N = \sqrt{||\mathbf{a}||^2 - a_T^2} = \sqrt{1 + \dfrac{4}{9t^{2/3}} - \dfrac{4t^{2/3}}{9(1+t^{4/3})}}$

7. $\mathbf{v}(t) = \mathbf{i} - \mathbf{j} + (1-32t)\mathbf{k}$, $||\mathbf{v}(t)|| = \sqrt{2+(1-32t)^2}$, $\mathbf{r}(t) = t\mathbf{i} - t\mathbf{j} + (t-16t^2)\mathbf{k}$

8. (a) $ds/dt = ||\mathbf{r}'(t)|| = 2t + \frac{1}{t}$
   (b) $L = 3 + \ln 2$

9. We have $\mathbf{v} = \mathbf{i} + t^{-1/2}\mathbf{k}$, $\|\mathbf{v}\| = \sqrt{1+t^{-1}}$, $\mathbf{a} = \frac{-1}{2}t^{-3/2}\mathbf{k}$, $\mathbf{v}\times\mathbf{a} = \frac{1}{2}t^{-3/2}\mathbf{j}$. So
$$\kappa(t) = \frac{\|\mathbf{v}\times\mathbf{a}\|}{\|\mathbf{v}\|^3} = \frac{t^{-3/2}}{2(1+t^{-1})^{3/2}}.$$
In particular, $\kappa(1) = \dfrac{1}{4\sqrt{2}}$ and $\rho(1) = \kappa(1)^{-1} = 4\sqrt{2}$.