Math 241 **Midterm #2** **Fall 2012**

Fri. Nov. 2nd

<u>**Instructions**</u>: Number the answer sheets from 1 to 5. Fill out **all** the information at the top of **each** sheet (write and sign the Honor Pledge on page 1 only). Answer **one** question on **each** sheet in the correct order: Problem 1 on sheet 1, problem 2 on sheet 2...(Use the back of the correct sheet if you need more space).

None of the following are allowed: lecture notes, book, electronic devices of any kind (including calculators, cell phones, etc.)

You may keep with you **one sheet of handwritten notes**.

1. Consider the function $f(x,y) = 2xe^{x^2-y^2}$.
   (a) (8pts) Find $\frac{\partial f}{\partial x}(1,1)$ and $\frac{\partial f}{\partial y}(1,1)$.
   (b) (10pts) Find an equation for the plane tangent to the surface $z = f(x,y)$ at the point $(1,1,2)$. Write your answer in the form $ax + by + cz = d$.
   (c) (7pts) Use the tangent plane approximation to approximate the value $f(1.02, 0.98)$. You do not need to simplify your answer.

2. (a) (10pts) Find the directional derivative of $f(x,y,z) = x^2 \sin(yz)$ at the point $(1, \pi/2, 2)$ in the direction of the vector $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$.
   (b) (10pts) Let $z = \ln(x-y)$ and let $x = s^2 + t^2$ and $y = st$. Compute $\frac{\partial z}{\partial s}$. Your answer need not be simplified, but it should be in terms of s and t only.

3. (a) (15pts) Find all critical points of $f(x,y) = x^3 - 6x^2 - 3y^2$ and determine whether each critical point yields a relative maximum, a relative minimum or a saddle point.
   (b) (15pts) Use the Lagrange multiplier method to find the extreme values of $f(x,y) = x^2 + y^2$ subject to the constraint $x^4 + y^4 = 1$.

4. (10pts) Evaluate the iterated integral $\int_0^1 \int_0^y \cos(x)\,dx\,dy$.

5. (15pts) Evaluate the double integral

$$\iint_R 3\,dA$$

where R is the region enclosed by the curves $y = x^2$ and $y = 4$.