Review problems for the second midterm

These are typical problems that could be on the exam, though there are more problems here than could possibly be on the exam (you can expect 4 or 5 problems on the exam).

The exam may include material that is not on the review and the wording of questions may be somewhat different. Make sure that you do all suggested homework problems and review all the material discussed in class.

1. Consider the function $z = x^{-1}e^{-y/x}$ for $x \neq 0$. Show that z satisfies the equation
$$\frac{\partial z}{\partial x} = y\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial y}.$$

2. Determine the following limit if it exists. If it does not exist, explain why.
$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

3. Find the directional derivative of $f(x, y) = 5x^2y$ at $(1, 1)$ in the direction of $\mathbf{i} - \mathbf{j}$.

4. Consider the function $f(x, y) = \sin(xy)$.
   (a) Find the direction in which f increases most rapidly at the point $(x_0, y_0) = (1, \pi)$.
   (b) What is the maximal directional derivative of f at this point?

5. Consider the function $f(x, y, z) = \sin(xy) + \cos(xz)$. Find an equation for the plane tangent to the level surface $f(x, y, z) = 0$ at the point $(x_0, y_0, z_0) = (\pi/4, 4, 2)$.

6. Consider the function $f(x, y) = -2x^2 + 3xy + y^2 - 4x + 3y - 1$.
   (a) Find all critical points of f.
   (b) Specify which of these points correspond to relative extreme values and saddle points of f.

7. Find the extreme value of $f(x, y) = (x - 1)^2 + y^2$ subject to the constraints $x^2 + y^2 \leq 4$ and $x \geq 0$.

8. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = xy + 2x$ on the curve $x^2 + y^2 = 4$.

9. Evaluate
$$\iint_D \frac{xy^2}{x^2+1}\, dA$$
where $D = \{(x, y)\,;\, 0 \leq x \leq 1\,,\ -3 \leq y \leq 3\}$.

10. Evaluate
$$\iint_D x \cos y \, dA$$
where D is the region bounded by the curves $y = 0$, $y = x^2$ and $x = 1$.

11. Evaluate the following integral:
$$\int_0^1 \int_y^1 \sin(x^2)\, dx\, dy.$$