Math 241 **Midterm #3** **Fall 2012**

Fri. Dec. 7th

<u>**Instructions**</u>: Number the answer sheets from 1 to 5. Fill out **all** the information at the top of **each** sheet (write and sign the Honor Pledge on page 1 only). Answer **one** question on **each** sheet in the correct order: Problem 1 on sheet 1, problem 2 on sheet 2...(Use the back of the correct sheet if you need more space).

None of the following are allowed: lecture notes, book, electronic devices of any kind (including calculators, cell phones, etc.)

You may keep with you **one sheet of handwritten notes**.

1. [20 pts] Consider the solid region D that is bounded below by the cone $z = \sqrt{x^2+y^2}$ and above by the sphere $x^2+y^2+z^2=4$. Evaluate the integral

$$\iiint_D 5z^2\,dV.$$

2. [15 pts] Evaluate the line integral $\int_{\mathcal{C}} y\,dx - x\,dy + xy\,dz$, where $\mathcal{C}$ is the curve parametrized by

$$\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k} \qquad \text{for } 0 \le t \le \frac{\pi}{2}.$$

3. [20 pts] By applying **Green's theorem**, evaluate the line integral

$$\int_{\mathcal{C}} (y+1)e^{x^2}\,dx + (yx^2+y^2)\,dy$$

where the curve $\mathcal{C}$ is the triangle with vertices $(0,0)$, $(1,0)$, $(1,1)$, oriented counterclockwise.

4. [20 pts] Let Σ be the part of the paraboloid $z = 16 - x^2 - y^2$ lying above the xy-plane. Let

$$\mathbf{F}(x,y,z) = \frac{x}{\sqrt{x^2+y^2}}\,\mathbf{i} + \frac{y}{\sqrt{x^2+y^2}}\,\mathbf{j} + 4\,\mathbf{k}.$$

Evaluate the integral

$$\iint_\Sigma \mathbf{F}\cdot\mathbf{n}\,dS$$

where $\mathbf{n}$ is the unit normal vector to Σ directed **upward**.

5. [25 pts] Let D be the region inside the cylinder $(x-1)^2+y^2=1$, below the paraboloid $z = 4 - x^2 - y^2$ and above the xy-plane. We denote by Σ the boundary of D, oriented positively (that is with the normal unit vector $\mathbf{n}$ pointing outward).

Let $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + xy\mathbf{k}$. Use the divergence theorem to compute

$$\iint_\Sigma \mathbf{F}\cdot\mathbf{n}\,dS.$$

You can use the formula

$$\int \cos^2\theta\,d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta), \qquad \int \cos^4\theta\,d\theta = \frac{3}{8}\theta + \frac{1}{4}\sin(2\theta) + \frac{1}{32}\sin(4\theta)$$