Solution to the review problems

1. (a) $I = \int_0^{2\pi} \int_0^2 \int_0^{7-r^2} r^4 \, dz \, dr \, d\theta$
   (b) $\frac{1856\pi}{35}$
2. $V = \int_0^{\pi/2} \int_{\cos\theta}^1 \int_0^{r\cos\theta + r\sin\theta + 5} r \, dz \, dr \, d\theta$
3. $\int_0^1 \int_0^u (u+17v)^\pi 17^2 \, dv \, du$
4. (a) $\vec{curl}\mathbf{F} = (x^2 - x^2)\mathbf{i} + (2xy - 2xy)\mathbf{j} + (2xz - 2xz)\mathbf{k} = \mathbf{0}$.
   (b) We have:
      - $f_x = 2xyz$, so $f(x,y,z) = x^2yz + g(y,z)$.
      - $f_y = x^2z = x^2z + g_y$ so $g(y,z) = h(z)$.
      - $f_z = x^2y + 1 = x^2y + h'(z)$, so $h(z) = z + C$.

      We get $f(x,y,z) = x^2yz + z + C$.
   (c) By the fundamental theorem of line integral, we get:
   $$\int_C \mathbf{F} \cdot d\mathbf{r} = f(\mathbf{r}(1)) - f(\mathbf{r}(-1)) = f(-1,1,1) - f(-1,-1,-1) = 2$$
5. $$I = \int_0^1 t^3 + 2t^5 - 3t^8 \, dt = \frac{1}{4}.$$
6. $4(x+8) + 32(z - \ln 2) = 0$
7. (a) $I = \int\int_R -4(x^2+y^2) \, dA$, where $R = \{(x,y)\,;\, x^2+y^2 \le 1\}$.
   (b)
      - $I = \int_0^{2\pi} \int_0^1 -4r^3 \, dr \, d\theta = -2\pi$
      - If we parametrize C by $x(t) = \cos t$, $y(t) = \sin t$ for $0 \le t \le 2\pi$. We get
      $$I = \int_0^{2\pi} \sin t(-\sin t) - \cos t \cos t \, dt = -2\pi.$$
8. (a) The surface is the graph of $f(x,y) = -2\sqrt{1-x^2-y^2}$ above the region $R = \{(x,y)\,;\, x^2+y^2 \le 1\}$. We thus have
   $$I = \int\int_R -Mf_x - Nf_y + P \, dA = \int\int_R -\frac{2x}{\sqrt{1-x^2-y^2}} - \frac{2y}{\sqrt{1-x^2-y^2}} + 2 \, dA$$
   (b) Using polar coordinates, we get
   $$I = \int_0^{2\pi} \int_0^1 \left[-\frac{2r\cos\theta + 2r\sin\theta}{\sqrt{1-r^2}} + 2 \right] r \, dr \, d\theta = \int_0^{2\pi} \int_0^1 2r \, dr \, d\theta = 2\pi$$
9. (a) since $\text{div}\mathbf{F} = 3x^2 + 3y^2 + 3z^2$ and D is the cube $\{(x,y,z)\,;\, 0 < x < 1,\ 0 < y < 1,\ 0 < z < 1\}$, we get
   $$I = \int\int\int_D 3x^2 + 3y^2 + 3z^2 \, dx \, dy \, dz = 3$$
   (b) We compute the integral on each sides of the cube (6 integrals). For instance, on the face $x = 0$, we have $\mathbf{n} = -\mathbf{i}$ and so $\mathbf{F} \cdot \mathbf{n} = -x^3 = 0$. So the integral is zero on that face.
   On the face $x = 1$, we have $\mathbf{n} = \mathbf{i}$ and so $\mathbf{F} \cdot n = x^3 = 1$. So the integral is 1 (the area of the side is 1).