Math 461

Midterm #2 - Solution to the practice exam

1. (a) True or False. Justify carefully your answer.

   i. True: $H = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ is a vector space.

   ii. False: $\vec{0}$ is not in S.

   iii. False (the Rank Theorem gives dim(Nul(A))= 4)

   iv. False: Only one of the three type of row operations preserve the determinant.

   (b) i. $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

   ii. $\left\{ \begin{bmatrix} 4 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \\ 0 \\ -4 \\ 1 \end{bmatrix} \right\}$

   iii. $\left\{ \begin{bmatrix} 1 \\ 0 \\ -4 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 4 \end{bmatrix} \right\}$

2. (a) $\det(A) = 4$, $\det(B) = 64$, $\det(C) = 4^4$.

   (b) $\begin{vmatrix} 3t & 2 \\ 9 & 2t \end{vmatrix} = 6(t^2 - 3)$, so the system has a unique solution for every $t \neq \pm\sqrt{3}$.

3. (a) i. H =Nul(A) where $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -5 \end{bmatrix}$, so H is a subspace.

   ii. $\left\{ \begin{bmatrix} -2 \\ 5 \\ 1 \end{bmatrix} \right\}$, $\dim(H) = 1$.

   i. G =Span$\left\{ \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \right\}$ is a subspace.

   ii. $\left\{ \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} \right\}$, $\dim(G) = 2$.

4. (a) i. $\vec{b_1}$ and $\vec{b_2}$ are linearly independent (not multiple of each other) and $\dim(\mathbb{R}^2) = 2$, so $\mathcal{B}$ is a basis.

ii. $[\vec{u}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(b) i. $\dim(P_2(\mathbb{R})) = 3$

ii. we just need to show that $\{[\vec{p}_1]_{\mathcal{B}}, [\vec{p}_2]_{\mathcal{B}}, [\vec{p}_3]_{\mathcal{B}}\}$ is linearly independent. (Row reduce the corresponding matrix).

iii. $[\vec{p}]_{\mathcal{C}} = \begin{bmatrix} 3 \\ 2 \\ -4 \end{bmatrix}$.

5. (a) $\lambda = 2$ and $\lambda = 3$.

(b) $\lambda = 10$: $\left\{ \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}$

$\lambda = 1$: $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \right\}$