Spring 2013 - Math 461

Midterm #3 - May 6th

- Write your name and section number **on every page**. (You only need to write and sign the honor pledge on the first page).
- This exam has **4 questions**. Do each question on a **different** answer sheet.
- Show enough work to justify your answers. **Unjustified answers will receive no credit.**
- **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

1. (a) [10pts] True or False. **Justify** your answers carefully.

   i. If a matrix A is orthogonally diagonalizable, then A is symmetric.

   ii. The solution of a least-squares problem is always unique.

   iii. If a $n \times n$ matrix has n distinct eigenvalues, then it is diagonalizable.

   iv. If two matrices are row equivalent, then they have the same eigenvalues.

   (b) [15pts] Let A be a 2×2 matrix given by $A = PDP^{-1}$ where

$$P = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} \text{ and } D = \begin{bmatrix} .5 & 0 \\ 0 & 2 \end{bmatrix}.$$

   Let $\vec{x}_k$ be solution of the dynamical system $\vec{x}_{k+1} = A\vec{x}_k$, with $\vec{x}_0$ such that

$$P^{-1}\vec{x}_0 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

   i. Compute $\vec{x}_1 = A\vec{x}_0$.

   ii. Find a formula for $\vec{x}_k$ for all k, involving the vectors $\vec{u_1} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\vec{u_2} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$ (which are the columns of P).

2. [25pts] Consider the matrix $A = \begin{bmatrix} 2 & -3 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

   (a) Show that the eigenvalues of A are $\lambda = 2$ and $\lambda = -1$.

   (b) Diagonalize A (give the matrices P and D. You do not need to compute P^{-1}).

3. (a) [15pts] Let $A = \begin{bmatrix} 1 & 2 \\ -3 & 3 \end{bmatrix}$. Find the (complex) eigenvalues and corresponding (complex) eigenvectors of A.

   (b) [10pts] Consider the quadratic form $Q(\vec{x}) = 3x_1^2 + x_2^2 + 4x_1x_2$. Find the symmetric matrix A such that $Q(\vec{x}) = \vec{x}^T A\vec{x}$ and classify the quadratic form Q as positive definite, negative definite or indefinite.

4. Let $W = \text{Span}\{\vec{u_1}, \vec{u_2}\}$ where $\vec{u_1} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$ and $\vec{u_2} = \begin{bmatrix} 4 \\ -6 \\ 1 \end{bmatrix}$.

(a) [10pts] Find an orthogonal basis of W.

(b) [10pts] Find the projection of the vector $\vec{b} = \begin{bmatrix} 10 \\ 5 \\ 9 \end{bmatrix}$ onto W.

(c) [5pts] Find the distance from $\vec{b}$ to W.