Math 461

Practice exam #3 - Solution

1. (a) False (the zero matrix is diagonalizable)
   (b) False (the identity matrix is diagonalizable but its only e-value is 1)
   (c) True (theorem)
   (d) True (theorem - this is the normal equation)

2. (a) $A\vec{v}_1 = -9\vec{v}_1$, so $\lambda_1 = -9$.
   (b) $A - 2I$ has null space $\text{Span}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\3\\1\end{bmatrix}\right\}$
   (c) $P = \begin{bmatrix}-1 & 1 & 0\\-2 & 0 & 3\\3 & 0 & 1\end{bmatrix}$, $D = \begin{bmatrix}-9 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{bmatrix}$.

3. (a) $\lambda_1 = 1 + i\sqrt{3}$, $v_1 = \begin{bmatrix}1\\-2+i\sqrt{3}\end{bmatrix}$
   $\lambda_2 = 1 - i\sqrt{3}$, $v_2 = \begin{bmatrix}1\\-2-i\sqrt{3}\end{bmatrix}$
   (b) $P = \begin{bmatrix}1 & 0\\-2 & -\sqrt{3}\end{bmatrix}$, $C = \begin{bmatrix}1 & -\sqrt{3}\\\sqrt{3} & 1\end{bmatrix}$
   (c) $r = 2$, $\varphi = \frac{\pi}{3}$.

4. $A = \begin{bmatrix}2 & 2\\2 & -1\end{bmatrix}$. Eigenvalues 3 and -2, so Q is indefinite.

5. $A^TA = \begin{bmatrix}2 & 2\\2 & 3\end{bmatrix}$ and $A^T\vec{b} = \begin{bmatrix}4\\8\end{bmatrix}$, so $\vec{x}_0 = \begin{bmatrix}-2\\4\end{bmatrix}$

6. $\vec{v}_1 = \begin{bmatrix}-1\\0\\1\\1\end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix}0\\2\\-1\\1\end{bmatrix}$ $\vec{v}_3 = \begin{bmatrix}1\\0\\1/2\\1/2\end{bmatrix}$

7. (a) $\text{Proj}_L(\vec{x}) = \begin{bmatrix}-1/2\\1/2\\0\end{bmatrix}$
   (b) $\text{Proj}_W(\vec{x}) = \vec{x} - \text{Proj}_L(\vec{x}) = \begin{bmatrix}3/2\\3/2\\3\end{bmatrix}$