Fall 2012 - Math 462
Partial Differential Equations for Scientists and Engineers
Homework #10 - Due Monday Nov. 12th

1. (20pts) Find the solution of the following IBVP:

$$\begin{aligned}
&u_{tt} - 4u_{xx} = \cos(t) \qquad 0 < x < \infty, \;\; t > 0\\
&u(x,0) = \sin(x), \qquad u_t(x,0) = 0,\\
&u(0,t) = \sin(t).
\end{aligned}$$

2. (30pts) Using the reflection method (with an even reflection), find a formula for the solution of the Neumann problem for the wave equation on the half-line:

$$\begin{aligned}
&u_{tt} - c^2 u_{xx} = 0 \qquad 0 < x < \infty, \;\; t > 0\\
&u(x,0) = \phi(x), \qquad u_t(x,0) = \psi(x),\\
&u_x(0,t) = 0.
\end{aligned}$$

3. (30pts) Consider the diffusion equation $u_t - ku_{xx} = 0$ on $(0,L)$ with Robin boundary conditions $u_x(0,t) - a_0 u(0,t) = 0$ and $u_x(L,t) + a_L u(L,t) = 0$. If $a_0 > 0$ and $a_L > 0$, use the energy method to show that the endpoints contribute to the decrease of the energy $\int_0^L u(x,t)^2\, dx$.

4. (20pts) Let u and v be such that

$$u_t - u_{xx} = f \quad \text{for } 0 < x < L, \; t > 0$$

and

$$v_t - v_{xx} = g \quad \text{for } 0 < x < L, \; t > 0$$

with $f(x,t) \leq g(x,t)$ for all x, t, and with $u \leq v$ at $x = 0$, $x = L$ and $t = 0$. Prove that $u(x,t) \leq v(x,t)$ for $x \in [0,L]$, $t > 0$.