**Fall 2012 - Math 462**
**Partial Differential Equations for Scientists and Engineers**
Homework #11 - Due Monday Nov. 26th

1. (25pt) The purpose of this exercise is to show that the maximum principle is not true for the equation $u_t = xu_{xx}$, which has a variable coefficient.

   (a) Verify that $u(x,t) = -2xt - x^2$ is a solution. Find the location of its maximum in the closed rectangle $\{-2 \le x \le 2\,,\ 0 \le t \le 1\}$. Why does this contradict the maximum principle?

   (b) Where precisely does the proof of the maximum principle break down for this equation?

2. (25pt) Find the formula for the solution of the diffusion equation with constant dissipation:

$$u_t - ku_{xx} + bu = 0 \qquad \text{for } -\infty < x < \infty$$

   with $u(x,0) = \phi(x)$, where $b > 0$ is a constant. (**Hint:** Make the change of variable $u(x,t) = e^{-bt}v(x,t)$).

3. (25pt) Find the formula for the solution of the diffusion equation with convection:

$$u_t - ku_{xx} + Vu_x = 0 \qquad \text{for } -\infty < x < \infty$$

   with $u(x,0) = \phi(x)$, where $V$ is a constant. (**Hint:** Substitute $y = x - Vt$.)

4. (25pt) Solve the following diffusion equation IBVP on the half line:

$$\begin{aligned} &u_t - ku_{xx} = 0 \qquad 0 < x < \infty, \quad t > 0 \\ &u(x,0) = e^{-x} \\ &u(0,t) = 0. \end{aligned}$$