

Fall 2012 - Math 462
Partial Differential Equations for Scientists and Engineers
Homework #12 - Due Monday Dec 3rd

1. (30 pts) Solve the following BVP in the rectangle $\{(x, y); 0 \leq x \leq 1, 0 \leq y \leq 1\}$:

$$\begin{cases} \Delta u = 0 & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ u_x(0, y) = 0 & 0 \leq y \leq 1, \quad u_x(1, y) = \cos(\pi y) + \cos(2\pi y) & 0 \leq y \leq 1 \\ u_y(x, 0) = 0 & 0 \leq x \leq 1, \quad u_y(x, 1) = 0 & 0 \leq x \leq 1 \end{cases}$$

2. (30 pts) Find the solution of the following BVP in the semi-infinite strip $\{(x, y); 0 \leq x \leq 1, y \geq 0\}$

$$\begin{cases} u_x(0, y) = 0, & u_x(1, y) = 0 & \text{for all } y \geq 0 \\ u(x, 0) = x & & \text{for all } 0 \leq x \leq 1, \end{cases}$$

with $u(x, y)$ bounded (that is $|u(x, y)| \leq M$ for some M).

3. (40 pts) Find the solution of Laplace's equation in the semi-infinite strip $\{(x, y); 0 \leq x \leq 2, y \geq 0\}$ satisfying the following mixed boundary conditions:

$$\begin{cases} u(0, y) = 0, & u_x(2, y) = 0 & \text{for all } y \geq 0 \\ u(x, 0) = 2 \sin(3\pi x/4) - 3 \sin(7\pi x/4) & & \text{for all } 0 \leq x \leq 2, \\ \lim_{y \rightarrow +\infty} u(x, y) = 0 & & \end{cases}$$