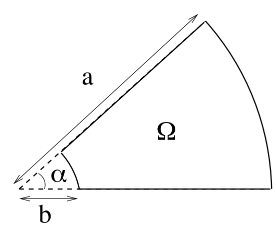
Fall 2012 - Math 462
Partial Differential Equations for Scientists and Engineers
Homework #13 - Not collected

1. Find the solution of $\Delta u = 0$ in the disk of radius 1, satisfying $u = 2\sin\theta - \sin(3\theta)$ on the boundary.

2. Consider a domain Ω obtained by taking a circular sector with angle α and radius a and cutting out a smaller circular sector of radius b:



Find the solution of the following BVP in Ω:

$$\begin{cases} \Delta u = 0 & \text{in } \Omega \\ u(r,0) = 0 & \text{for } b < r < a, \qquad u(r,\alpha) = 0 \quad \text{for } b < r < a \\ u(b,\theta) = 0 & \text{for } 0 < \theta < \alpha, \qquad u(a,\theta) = f(\theta) \quad \text{for } 0 < \theta < \alpha \end{cases}$$

3. Let $\Omega = \{(x,y)\,;\, 1 < x^2 + y^2 < 4\}$. Find u such that

$$\Delta u = 0 \text{ in } \Omega$$

$$u = 0 \text{ on } \{(x,y)\,;\, x^2 + y^2 = 1\}$$
$$u = 1 \text{ on } \{(x,y)\,;\, x^2 + y^2 = 4\}.$$

4. Let Ω be the half disk $\Omega = \{(x,y)\,;\, x^2 + y^2 \le 4\,,\ y > 0\}$. Find u such that

$$\Delta u = 0 \text{ in } \Omega$$

$$u = 0 \text{ on } \{(x,y)\,; y = 0, -2 < x < 2\}$$
$$u = f \text{ on } \{(x,y)\,;\, x^2 + y^2 = 4,\ y > 0\}.$$

for some function f.