Fall 2012 - Math 462
Partial Differential Equations for Scientists and Engineers
Homework #2 - Due Monday Sept 17th

1. (20 pts)
   (a) Solve the equation $y\,u_x + x\,u_y = 0$ with $u(0,y) = e^{-y^2}$.
   (b) Sketch the characteristic curves.
   (c) In which region of the xy plane is the solution uniquely determined?

2. (20 pts) Consider the initial value problem

$$\begin{aligned}&(1+t)\,u_t + x\,u_x = (1+t)\,u^2\\&u(x,0) = \sin(x)\end{aligned}$$

   (a) Find the equation for the characteristic curves $x(t)$.
   (b) Find the solution $u(x,t)$.
   (c) Verify that the function you obtained in (b) is a solution of the initial value problem.

3. (20 pts) Solve $u_x + u_y + u = e^{2+2y}$ with $u(x,0) = 0$.

4. (20 pts) Carefully derive the equation of a string oscillating in a medium in which the resistance is proportional to the velocity (there is an additional force that you must take into account, which is in the direction opposite to the motion of the string, and with modulus $\gamma|u_t|$ for some constant γ).

5. (20 pts) Find the values of λ for which the following boundary value problem has non trivial solutions:

$$X'' + \lambda X = 0 \quad \text{for } 0 < x < L, \qquad X'(0) = 0, \quad X'(L) = 0.$$

   For each such λ, find the corresponding solutions $X(x)$.