Fall 2012 - Math 462
Partial Differential Equations for Scientists and Engineers
Homework #3 - Due Monday Sept. 24

1. (25pt) Consider waves in a resistant medium that satisfy the problem

$$u_{tt} - c^2 u_{xx} = -r u_t \quad 0 < x < L, \quad t > 0$$
$$u(0,t) = 0, \quad u(L,t) = 0$$

where r is a constant $0 < r < 2\pi c/L$. Use the separation of variables method to find a series solution of this boundary value problem.

2. (a) (25pt) Find the values of λ for which the following boundary value problem has non trivial solutions:

$$X'' + \lambda X = 0 \quad \text{for } 0 < x < 1, \qquad X(0) = 0, \quad X'(1) = 0.$$

For each such λ, find the corresponding solutions $X(x)$.

(b) (25pt) Use the separation of variables method and your answer to the question above to find the solution of the following **mixed** boundary problem:

$$u_t - k u_{xx} = 0 \quad 0 < x < 1, \quad t > 0$$
$$u(0,t) = 0, \quad u_x(1,t) = 0$$
$$u(x,0) = \sin(\tfrac{3\pi}{2}x) - 2\sin(\tfrac{5\pi}{2}x)$$

3. (25pt) Find the eigenvalues graphically for the boundary problem

$$X'' + \lambda X = 0 \qquad 0 < x < L$$

$$X(0) = 0 \qquad X'(L) + X(L) = 0$$