Fall 2012 - Math 462
Partial Differential Equations for Scientists and Engineers
Homework #5 - Due Monday Oct. 8

1. (20pt) For each of the following functions, state whether it is even or odd or periodic. If periodic, what is the smallest period?

   (a) $\tan(ax)$ $(a > 0)$

   (b) $\sin(x^2)$

   (c) e^{-x}

   (d) x^m $(m =\text{integer})$

   (e) $\dfrac{\sin(x)}{2 + \cos(x)}$

2. (30) Compute the full Fourier series of the function $f(x) = |x|$ on the interval $(-L, L)$. Sketch the function to which the Fourier series converges on the interval $(-3L, 3L)$.

3. (25pt)

   (a) Let $\phi(x)$ be a continuous function on $(0, L)$. Under what conditions is its **odd** extension also a continuous function?

   (b) Same as part (a) for the **even** extension.

4. (25pt) We saw in Homework #3 that the boundary conditions $X(0) = 0$, $X'(1) = 0$ lead to the eigenfunctions

$$X_n(x) = \sin\left(\left(n + \frac{1}{2}\right)\pi x\right) \qquad n = 0, 1, 2, \ldots$$

Assuming that a function $\phi(x)$ defined on $(0, 1)$ can be written as

$$\phi(x) = \sum_{n=0}^{\infty} c_n \sin\left(\left(n + \frac{1}{2}\right)\pi x\right)$$

find a formula for the coefficient c_n as an integral of ϕ.