Fall 2012 - Math 462
Partial Differential Equations for Scientists and Engineers
Homework #6 - Due Monday Oct. 15

1. (30pts) Let γ_n be a sequence of constants tending to ∞. Let $f_n(x)$ be a sequence of functions defined as follows:

   $f_n(1/2) = 0$

   $f_n(x) = \gamma_n$ in the interval $[\frac{1}{2} - \frac{1}{n}, \frac{1}{2})$

   $f_n(x) = -\gamma_n$ in the interval $(\frac{1}{2}, \frac{1}{2} + \frac{1}{n}]$

   $f_n(x) = 0$ elsewhere.

   Show that:

   (a) $f_n(x)$ converges to 0 pointwise.

   (b) The convergence is not uniform.

   (c) $f_n(x)$ converges to 0 in the L^2 sense if $\gamma_n = n^{1/3}$.

   (d) $f(x)$ does not converge in the L^2 sense if $\gamma_n = n$.

2. (30pts) Let

$$\phi(x) = \begin{cases} -1 - x & \text{for } -1 < x < 0 \\ 1 - x & \text{for } 0 < x < 1. \end{cases}$$

   (a) Find the full Fourier series of $\phi(x)$ in the interval $(-1, 1)$.

   (b) Does it converge in the mean square sense?

   (c) Does it converge pointwise?

   (d) Does it converge uniformly to $\phi(x)$ in the interval (-1,1)?

3. (20pts) Let $f(x) = \cosh(x)$, $-\pi \leq x \leq \pi$.

   (a) Find the full Fourier series of f (recall that $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$).

   (b) For which values of x does the Fourier series of f converges to f?

   (c) Use (b) to evaluate the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$.

4. (20pts) Find the solution of the following IBVP:

$$\begin{aligned} &u_{tt} - u_{xx} = 0 \qquad\qquad 0 < x < 1 \quad t > 0 \\ &u(0,t) = 0, \quad u(1,t) = 0 \\ &u(x,0) = x, \quad u_t(x,0) = 1 \end{aligned}$$