Fall 2012 - Math 462
Partial Differential Equations for Scientists and Engineers
Homework #7 - Due Monday Oct. 22

1. (25pts) Find the solution of the following non-homogeneous IBVP:

$$\begin{aligned} &u_{tt} - u_{xx} = t\sin(\pi x) \qquad 0 < x < 1 \quad t > 0 \\ &u(0,t) = 0, \quad u(1,t) = 0 \\ &u(x,0) = x, \quad u_t(x,0) = 0 \end{aligned}$$

2. (25pts) Find the solution of the following non-homogeneous IBVP:

$$\begin{aligned} &u_t - u_{xx} = 0 \qquad 0 < x < 1 \quad t > 0 \\ &u(0,t) = t, \quad u(1,t) = 0 \\ &u(x,0) = x, \end{aligned}$$

3. (25pts) Find d'Alembert's solution for the following wave equation problem on the whole line:

$$\begin{aligned} &u_{tt} - u_{xx} = 0 \qquad -\infty < x < \infty \quad t > 0 \\ &u(x,0) = \phi(x), \quad u_t(x,0) = 0 \end{aligned}$$

where $\phi(x) = \begin{cases} 1 & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$. Sketch the solution for $t = 0$, $t = 1/2$, $t = 1$, $t = 2$.

4. (25pts) Solve

$$u_{xx} - 3u_{xt} - 4u_{tt} = 0, \quad -\infty < x < \infty \quad t > 0$$

with initial conditions

$$u(x,0) = x^2\,, \qquad u_t(x,0) = 0.$$

(Hint: Factor the operator as we did for the wave equation and show that the general solution of the PDE is $f(4x+t) + g(x-t)$.)