Fall 2012 - Math 462
Partial Differential Equations for Scientists and Engineers
Homework #8 - Due Monday Oct. 29

1. (15pts) Solve the following initial value problem:

$$u_{tt} - c^2 u_{xx} = 0 \qquad \text{for } -\infty < x < +\infty,\ t > 0$$
$$u(x,0) = 0 \quad \text{for all } x$$
$$u_t(x,0) = \frac{1}{1+x^2} \quad \text{for all } x$$

2. (15pts) Solve the following initial value problem:

$$u_{tt} - 25u_{xx} = 0 \qquad \text{for } -\infty < x < +\infty,\ t > 0$$
$$u(x,0) = x^2 \quad \text{for all } x$$
$$u_t(x,0) = 3 \quad \text{for all } x$$

3. (20pts) The midpoint of a piano string of tension T, density ρ (we recall the formula $c = \sqrt{T/\rho}$) and length ℓ is hit by a hammer whose head diameter is $2a$. A flea is sitting on the string at a distance $\ell/4$ from one end. How long does it take for the disturbance to reach the flea? (assuming $a < \ell/4$).

4. (20pts) Consider the damped string equation:

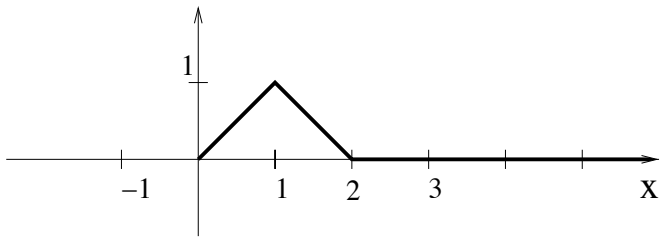
$$u_{tt} - c^2 u_{xx} + \gamma u_t = 0.$$

Show that the total energy decreases.

5. (30pts) Let $u(x,t)$ be the solution of the following wave equation on the **half-line**:

$$u_{tt} - u_{xx} = 0 \quad 0 < x < \infty,\ \ t > 0$$
$$u(x,0) = \phi(x)$$
$$u_t(x,0) = 0$$
$$u(0,t) = 0$$

with ϕ given by:



Sketch $u(x,t)$ for $t = 1$ and $t = 2$. Describe the behavior of u as t increases.