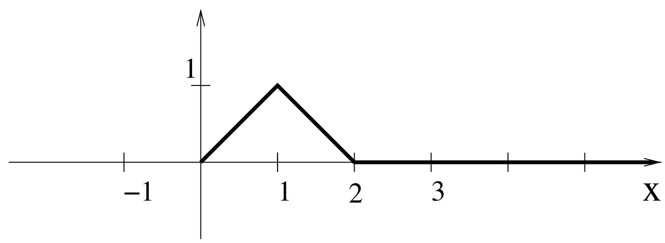
Fall 2012 - Math 462
Partial Differential Equations for Scientists and Engineers
Homework #9 - Due Monday Nov. 5th

1. (30pt) Let $u(x,t)$ be the solution of

$$\begin{array}{ll} u_{tt} - 4u_{xx} = 0 & 0 < x < \infty, \;\; t > 0 \\ u(x,0) = \phi(x), & u_t(x,0) = \psi(x), \\ u(0,t) = 0 & \end{array}$$

with ϕ given by



and $\psi = \begin{cases} 1 & \text{if } 1 < x < 3 \\ 0 & \text{otherwise} \end{cases}$.

Using the domain of influence, determine

(a) The time at which $u(10,t)$ becomes non zero for the first time.

(b) The time after which you are sure that $u(10,t)$ will always be zero.

You do not have to solve the PDE.

2. (40pt) Let $f(x,t)$ be any function and let

$$u(x,t) = \frac{1}{2c}\iint_\Delta f = \frac{1}{2c}\int_0^t \int_{x-ct+cs}^{x+ct-cs} f(y,s)\,dy\,ds$$

where Δ is the usual triangle of dependence of (x,t). Verify directly by differentiation that

$$u_{tt} - c^2 u_{xx} = f, \qquad u(x,0) = 0, \quad u_t(x,0) = 0.$$

3. (30pt) Find the solution of

$$\begin{array}{ll} u_{tt} - c^2 u_{xx} = e^x & -\infty < x < \infty, \;\; t > 0, \\ u(x,0) = 0, & \\ u_t(x,0) = \cos(x). & \end{array}$$