Fall 2009 - Math 463 Section 0201
Complex Variables for Scientists and Engineers
Homework #1 - Due Tuesday 15th **in class**

1. (a) Give the real part and imaginary part of the following complex numbers:

$$-3+9i, \quad -2i, \quad 5, \quad \overline{-5i+3}, \quad \overline{1+2i+\overline{(3-i)}}$$

   (b) Let $z_1 = 3+2i$ and $z_2 = 4-2i$. Give the real part and imaginary part of the following complex numbers:

$$z_1+z_2, \quad z_1-z_2, \quad z_1z_2, \quad z_1^{-1}, \quad \frac{z_2}{z_1}.$$

2. Verify that each of the two numbers $z = 1+i$ and $z = 1-i$ satisfies the equation $z^2 - 2z + 2 = 0$.

3. Plot the following points on a graph:

$$z_1 = -3i, \quad z_2 = 1+2i, \quad \overline{z_2}, \quad z_1+z_2, \quad 2z_2.$$

4. Describe and sketch the set of points determined by the following conditions (one graph for each):

   (a) $|z-4+3i| = 3$

   (b) $|z| \leq 3$

   (c) $\mathrm{Re}(\overline{z}+2+i) = 1$

5. Show that for any complex number z we have $|\overline{z}| = |z|$ (Hint: Use the formula $|z|^2 = z\overline{z}$).

6. Find the real part and imaginary part of the following complex numbers:

$$4e^{i\frac{\pi}{4}}, \quad 3e^{-i\frac{\pi}{2}}, \quad 2e^{i\frac{5\pi}{3}}.$$

7. (a) Give the modulus and principal argument of the following complex numbers. Then give their exponential form:

$$2-2i, \quad -4i, \quad -5, \quad -2\sqrt{3}-2i.$$

   (b) Use your answer above to find the real part and imaginary part of the following complex numbers

$$(2-2i)^5, \quad \frac{1}{2-2i}, \quad \frac{i}{2-2i}.$$

8. Use the exponential form of complex numbers to give a parametric representation of the circle of radius 5 centered at the point $(1, 3)$.

9. Use the exponential form to show that

$$(-1+i)^7 = -8(1+i).$$

10. Find all the roots (first in exponential form, then give the real part and imaginary part) of

$$(-8)^{1/3}, \quad (4i)^{1/3}, \quad (1-\sqrt{3}i)^{1/2}.$$