Fall 2009 - Math 463 Section 0201
Complex Variables for Scientists and Engineers
Homework #10 - Due Tuesday December 1st **in class**

1. Expand the given function in a Laurent series in the given annular domain
   (a) $f(z) = \dfrac{\cos z}{z}$, $0 < |z|$
   (b) $f(z) = \dfrac{e^z}{z-1}$, $0 < |z-1|$
   (c) $f(z) = z \cos \frac{1}{z}$, $0 < |z|$
2. Expand the function $f(z) = \frac{1}{z(z-3)}$ in a Laurent series in the given annular domain
   (a) $0 < |z| < 3$
   (b) $0 < |z-3| < 3$
   (c) $1 < |z-4| < 4$
3. Expand the function $f(z) = \frac{1}{z(z-1)^2}$ in a Laurent series in the annular domain $|z| > 1$.
4. Use an appropriate Laurent series to find the indicated residue
   (a) $f(z) = \dfrac{4z-6}{z(2-z)}$, $\mathrm{Res}_{z=0} f(z)$
   (b) $f(z) = (z+3)^2 \sin\left(\frac{2}{z+3}\right)$, $\mathrm{Res}_{z=-3} f(z)$
   (c) $f(z) = e^{-2/z^2}$, $\mathrm{Res}_{z=0} f(z)$
5. Determine the order of the poles for the given function
   (a) $f(z) = 5 - \dfrac{6}{z^2}$
   (b) $f(z) = \dfrac{e^z - 1}{z^5}$
6. Use any method to find the residue at each pole of the given function
   (a) $f(z) = \dfrac{z}{z^2+16}$
   (b) $f(z) = \dfrac{\cos z}{z^2(z-\pi)^3}$
7. Show that $z = 0$ is a removable singularity of the function $f(z) = \dfrac{e^{2z}-1}{z}$.
8. Use Cauchy's residue theorem to evaluate the given integral
   (a) $\displaystyle\int_C \frac{1}{(z-1)(z+2)^2}\, dz$ where C is the contour $|z| = 3/2$
   (b) $\displaystyle\int_C \frac{1}{(z-1)(z+2)^2}\, dz$ where C is the contour $|z| = 3$
   (c) $\displaystyle\int_C z^3 e^{-1/z^2}\, dz$ where C is the contour $|z+i| = 2$