Fall 2009 - Math 463 Section 0201
Complex Variables for Scientists and Engineers
Homework #2 - Due Thursday September 17th **in class**

1. Give the modulus, the principal argument, the real part and imaginary part of the following complex numbers:
$$(2-2i)^3, \quad (4i)^{-3}, \quad \frac{-2}{1-\sqrt{3}i}$$
2. Let $\omega_3 = e^{i\frac{2\pi}{3}}$ and define $f(z) = \omega_3 z$. What type of geometric transformation is f (find $|f(z)|$ and $\arg(f(z))$ in terms of $|z|$ and $\arg(z)$)?
3. Describe and sketch the set of points determined by the following conditions (one graph for each):
   (a) $|z-3i| \geq 1$
   (b) $z \neq 0$ and $-\pi < \mathrm{Arg}(z) < \pi$
   (c) $\mathrm{Re}(z) < 1/2$
   (d) $\mathrm{Im}(z) = 1$
4. Which sets in Exercise 3 are open? Which sets are closed?
5. For each set in Exercise 3, describe the interior, the closure and the boundary.
6. For each of the following functions, describe the domain of definition:
$$f(z) = \mathrm{Arg}(z^{-1}), \quad f(z) = \frac{1}{z^2+1}, \quad f(z) = \frac{1}{|z|^2+1}, \quad f(z) = \frac{z}{z+\bar{z}}$$
7. Write the function $f(z) = z^2 + z + 1$ in the form $f(z) = u(x,y) + iv(x,y)$.
8. Suppose that $f(x+iy) = x^2 - y + ixy$. Write $f(z)$ in terms of z (Hint: use the relations $\mathrm{Re}(z) = \frac{z+\bar{z}}{2}$ and $\mathrm{Im}(z) = \frac{z-\bar{z}}{2i}$)
9. Write the function $f(z) = \dfrac{z}{\bar{z}}$ in the form $f(z) = u(r,\theta) + iv(r,\theta)$.
10. Show that the line $2y = x$ is mapped onto a spiral under the transformation $f(z) = e^z$.
11. Sketch the region onto which the sector $r \leq 1$, $0 \leq \theta \leq \pi/4$ is mapped by the transformation $f(z) = z^2$.