Fall 2009 - Math 463 Section 0201 Complex Variables for Scientists and Engineers Homework #4 - Due Thursday October 1st in class

- 1. Find f'(z) for the given functions:
 - (a) $f(z) = (2-i)z^5 + iz^3 7z^2$
 - (b) $f(z) = \frac{iz^2 2z}{3z + 1 i}$
 - (c) $f(z) = (5z^2 z)^6$
 - (d) $f(z) = e^{(2+i)z-1}$
- 2. Show that the function f(z) = |z| is nowhere differentiable.
- 3. Show that the following functions are not analytic at any point:
 - (a) f(z) = y + ix(b) $f(z) = \overline{z}^2$
- 4. Show that the following functions are entire and find their derivatives:

(a)
$$f(z) = e^{x^2 - y^2} \cos(2xy) + ie^{x^2 - y^2} \sin(2xy)$$

(b) $f(z) = 4x^2 + 5x - 4y^2 + 9 + i(8xy + 5y - 1)$

- 5. Find the constant a and b such that the function f(z) = 3x y + 5 + i(ax + by 3) is entire.
- 6. Is the function $f(z) = \operatorname{Arg}(z)$ differentiable at any point in z?
- 7. Show that the function $f(z) = x^2 + y^2 + 2ixy$ is differentiable along the real axis. Is f analytic at any point?
- 8. Find the set of points at which the following function is differentiable:

$$f(z) = x^{2} - x + y + i(y^{2} - 5y - x)$$

- 9. Suppose that f is analytic. Show that $|f'(z)|^2 = u_x^2 + v_x^2$.
- 10. Verify that the following functions are harmonic in an appropriate domain. Find the harmonic conjugate of u and form the corresponding analytic function f(z) = u + iv:
 - (a) u(x,y) = 2x 2xy
 - (b) $u(x,y) = -e^{-x} \sin y$
 - (c) $u(x,y) = x^3 3xy^2$