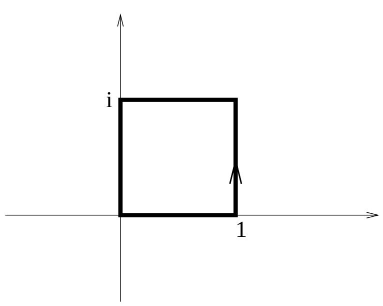
Fall 2009 - Math 463 Section 0201
Complex Variables for Scientists and Engineers
Homework #6 - Due Thursday October 22nd **in class**

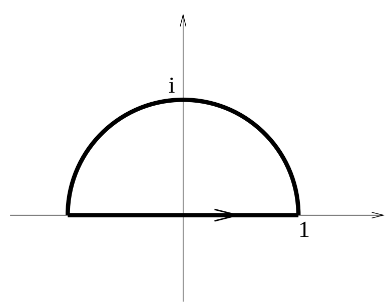
1. Compute the following integral:
   (a) $\int_0^2 3t^2 - it + 4i\,dt$
   (b) $\int_0^\pi e^{(1+i)t}\,dt$
   (c) $\int_0^{\pi/2} i\sin(it)\,dt$

2. Let $w(t)$ be a continuous complex valued function defined on an interval $-a \le t \le a$. Suppose that w is even (that is $w(-t) = w(t)$). Show that
$$\int_{-a}^{a} w(t)\,dt = 2\int_0^a w(t)\,dt$$

3. (a) Sketch the arc described by
$$z(t) = \begin{cases} t & 0 \le t \le 1 \\ 1 + i(t-1) & 1 < t \le 2 \\ (1+i)(t-1) & 2 < t \le 3 \end{cases}$$
   (b) Is this arc simple? Is it a Jordan curve?
   (c) Is this arc smooth? Is it a contour?

4. Find a parametrization of the following simple closed arcs:





5. Evaluate the given integral along the indicated contour
   (a) $\int_C (z+3)\,dz$, where C is $z(t) = 2t + i(4t-1)$, $1 \le t \le 3$.
   (b) $\int_C (2\overline{z} - z)\,dz$, where C is $z(t) = -t + i(t^2+2)$, $0 \le t \le 2$.
   (c) $\int_C \mathrm{Re}(z)\,dz$, where C the circle $|z| = 1$.

(d) $\int_C 2x - iy\, dz$, where C the lower half of the circle $|z| = 1$ from $z = -1$ to $z = 1$.

6. Evaluate the given integral along the indicated contour

   (a) $\int_C e^z\, dz$ where C is the polygonal path consisting of the line segments from $z = 0$ to $z = 2$ and from $z = 2$ to $z = 1 + \pi i$.

   (b) $\int_C 3z^{1/2}\, dz$ where $z^{1/2}$ is the principal branch of the square root function and C is the line segment between $z_1 = 1$ and $z_2 = 9i$.

   (c) $\int_C \frac{z+1}{z}\, dz$, where C the **right** half of the circle $|z| = 1$ from $z = -i$ to $z = i$.

   (d) $\int_C \frac{1}{z}\, dz$ where C the **left** half of the circle $|z| = 1$ from $z = -i$ to $z = i$

7. If a contour C has a parametric representation $z(t)$, $a \leq t \leq b$, then $-C$ has parametric representation $z(-t)$, $-b \leq t \leq -a$. Using the definition of the contour integral, show that

$$\int_C f(z)\, dz = -\int_{-C} f(z)\, dz$$

8. Find an upper bound for the modulus of the given integrals

   (a) $\int_C (z^2 + 4)\, dz$ where C is the line segment from $z = 0$ to $z = 1 + i$.

   (b) $\int_C \frac{1}{z^2 - 2i}\, dz$ where C is the right half of the circle $|z| = 3$ from $-3i$ to $z = 3i$.