Fall 2009 - Math 463 Section 0201
Complex Variables for Scientists and Engineers
Homework #7 - Due Thursday October 29th **in class**

1. Show that $\int_C f(z)\,dz = 0$ for the following functions f and when C is the unit circle $|z| = 1$:
   (a) $f(z) = z^3 - 1 + 3i$
   (b) $f(z) = \frac{z}{2z+3}$
   (c) $f(z) = z^2 + \frac{1}{z-4}$
   (d) $f(z) = \tan z$

2. Use Cauchy's theorem and what you know about integrals of the form $\int_C \frac{1}{(z-z_0)^n}\,dz$ to evaluate the given integrals:
   (a) $\int_C z + \frac{1}{z}\,dz$, where C is the circle $|z| = 2$ oriented positively.
   (b) $\int_C z + \frac{1}{z^2}\,dz$, where C is the circle $|z| = 2$ oriented positively.
   (c) $\int_C \frac{2z+1}{z^2+z}\,dz$, where C is the circle $|z| = 2$ oriented positively.
   (d) $\int_C \frac{(z-1)}{z(z-i)(z-3i)}\,dz$, where C is the circle $|z-i| = \frac{1}{2}$ oriented positively.
   (e) $\int_C \mathrm{Log}(z+10)\,dz$, where C is the circle $|z| = 2$ oriented positively.

3. Evaluate
$$\int_C \frac{e^z}{z+3} - 3\overline{z}\,dz$$
where C is the unit circle $|z| = 1$ oriented positively.

4. Use Cauchy's Formulas to evaluate the following integrals:
   (a) $\int_C \frac{4}{z-3i}\,dz$, where C is the circle $|z| = 5$ oriented positively.
   (b) $\int_C \frac{z^2-3z+4i}{z+2i}\,dz$, where C is the circle $|z| = 3$ oriented positively.
   (c) $\int_C \frac{z^2}{z^2+4}\,dz$, where C is the circle $|z-i| = 2$ oriented positively.
   (d) $\int_C \frac{\cos(2z)}{z^5}\,dz$, where C is the circle $|z| = 1$ oriented positively.
   (e) $\int_C \frac{z+2}{z^2(z-1-i)}\,dz$, where C is the circle $|z| = 1$ oriented positively.

5. Evaluate
$$\int_C \frac{\sin z}{(z-2i)^2} + \mathrm{Re}(z)\,dz$$
where C is the circle $|z| = 3$ oriented positively.