

**Fall 2009 - Math 463 Section 0201**  
**Complex Variables for Scientists and Engineers**  
**Homework #8 - Due Thursday November 5th in class**

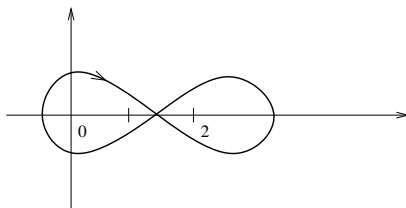
1. Evaluate the following complex integrals (all simple closed contours are oriented positively)

- (a)  $\int_C z e^z dz$  where  $C$  is the circle  $|z - i| = 1$ .
- (b)  $\int_C \frac{z e^z}{(z - i)^2} dz$  where  $C$  is circle  $|z| = 2$ .
- (c)  $\int_C e^z dz$  where  $C$  is the straight line from  $-i$  to  $2 + i$ .
- (d)  $\int_C \frac{1}{z^2(z^2 + 1)} dz$  where  $C$  is the circle  $|z - i| = \frac{3}{2}$ .
- (e)  $\int_C x - i y^2 dz$  where  $C$  is the straight line from  $0$  to  $1 + i$ .
- (f)  $\int_C \left( \frac{e^{2iz}}{z^4} + \frac{z^4}{(z - i)^3} \right) dz$  where  $C$  is the circle  $|z| = 6$ .

2. Evaluate the integral

$$\int_C \frac{3z + 1}{z(z - 2)^2} dz$$

where  $C$  is the following figure-eight contour:



3. Find the limit (if it exists) of the following sequences:

- (a)  $z_n = \frac{3+ni}{n+2ni}$
- (b)  $z_n = 5i^n$
- (c)  $z_n = \left( \frac{1+i}{4} \right)^n$  (Hint: You can use polar coordinates).

4. Determine whether the given series is convergent or divergent. If convergent, find its sum.

- (a)  $\sum_{n=0}^{\infty} (1 - i)^n$
- (b)  $\sum_{n=0}^{\infty} \left( \frac{i}{2} \right)^n$
- (c)  $\sum_{n=0}^{\infty} \frac{1}{2} i^n$
- (d)  $\sum_{n=0}^{\infty} 3 \left( \frac{2}{1 + 2i} \right)^n$