Fall 2009 - Math 463 Section 0201
Complex Variables for Scientists and Engineers
Homework #9 - Due Thursday November 19th **in class**

1. Find the radius and circle of convergence of the given power series:
   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(1-2i)^{n+1}}(z-2i)^n$
   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n2^n}(z-1-i)^n$
   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{4}{(2n)^n} z^n$

2. Find the first three nonzero terms of the Maclaurin series of $f(z) = \tan z$ (use Taylor's formula).

3. Use known results to find the Maclaurin series of the given function and give its radius of convergence
   (a) $f(z) = \dfrac{z}{1+z}$
   (b) $f(z) = \sin(z^2)$
   (c) $f(z) = ze^{-z^2}$
   (d) $f(z) = \dfrac{z}{(1-z)^3}$

4. Without computing the Taylor series, determine the radius of convergence of the Taylor series of the function
$$f(z) = \frac{4+z}{1+z^2}$$
centered at $z_0 = 2+5i$.

5. Use known results to find the Taylor series of the given function centered at z_0 and give its circle of convergence (remember that you can write $z = (z-z_0)+z_0$ if needed)
   (a) $f(z) = 1/z$ at $z_0 = 1$
   (b) $f(z) = \dfrac{1}{3-z}$ at $z_0 = 2i$
   (c) $f(z) = e^z$ at $z_0 = 3i$

6. Consider the function $f(z) = \text{Log}(1+z)$.
   (a) In what domain is f analytic? What is the radius of convergence of its Maclaurin series?
   (b) Using the fact that $f'(z) = \dfrac{1}{1+z}$ wherever f is analytic, find the Maclaurin series of f. What is the radius of convergence of this series
   (c) Using your result from part (b), find the Maclaurin series for $\text{Log}(1-z)$.