1. Let T be the linear transformation whose matrix relative to the standard basis $\mathcal{E}$ is
$$A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}$$
   (a) Find the matrix B of T relative to the basis $\mathcal{B} = \{(1,1),(2,1)\}$ by computing T applied to $(1,1)$ and T applied to $(2,1)$.

   (b) Find the matrix B of T relative to the basis $\mathcal{B}$ by using the change of basis formula
$${}_{\mathcal{B}}[T]_{\mathcal{B}} = P_{\mathcal{B}\leftarrow\mathcal{E}} \bullet {}_{\mathcal{E}}[T]_{\mathcal{E}} \bullet P_{\mathcal{E}\leftarrow\mathcal{B}}$$
   (c) Find the coordinates of the standard basis vector $(1,0)$ in terms of the basis $\mathcal{B}$ using the change of basis formula
$$[v]_{\mathcal{B}} = P_{\mathcal{B}\leftarrow\mathcal{E}}[v]_{\mathcal{E}}$$
   (d) Put $P = P_{\mathcal{B}\leftarrow\mathcal{E}}$, so $B = PAP^{-1}$. Show
$$B^n = PA^nP^{-1}.$$
   (To get full credit you should use mathematical induction).

   (e) Compute A^4. (Use the formula from (d))

2. (a) Show the only 2×2 matrices that commute with all diagonal 2×2 matrices are the diagonal 2×2 matrices.

   (b) Show the only 2×2 matrices that commute with all 2×2 matrices are scalar matrices (matrices of the form $\lambda I = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$). Hint: use part (a).

   In fact, this is true for all n.

3. Suppose $T : V \to W$ is a linear transformation, $\mathcal{A}$ and $\mathcal{B}$ are bases in V and $\mathcal{C}$ and $\mathcal{D}$ are bases in W. State **and prove** the relation between the matrix of T relative the bases $\mathcal{A}$ and $\mathcal{C}$ and the matrix of T relative to the bases $\mathcal{B}$ and $\mathcal{D}$. In other words, what is the relation between ${}_{\mathcal{C}}[T]_{\mathcal{A}}$ and ${}_{\mathcal{D}}[T]_{\mathcal{B}}$?

   (Your answer should involve the change of basis matrices like $P_{\mathcal{B}\leftarrow\mathcal{A}}$ and $P_{\mathcal{D}\leftarrow\mathcal{C}}$ and their inverses.)