Math 405 **Selected HW1 Solutions** 9/10/14

Selected HW1 Solutions

1. 2.4 e)

   These three vectors are linearly dependent. We give an explicit dependence relation

   $$(-1) < 1,1,2 > +(1) < 3,1,2 > +(2) < -1,0,0 >= \mathbf{0}$$

2. 2.5

   Let $\mathbf{u} =< 1,1,0 >$ and $\mathbf{v} =< 0,1,1 >$. We claim $S = S(\mathbf{u},\mathbf{v})$ is the solution set, call it X, of

   $$x_1 - x_2 + x_3 = 0. \tag{1}$$

   It is clear that $S \subset X$. Now suppose $X \not\subset S$. Then there is some vector $\mathbf{w} \in X$ with $w \notin S$. Since X is a subspace we have $S(\mathbf{u},\mathbf{v},\mathbf{w}) \subset X$. But, since $\mathbf{w} \notin S$ we have $S(\mathbf{u},\mathbf{v},\mathbf{w}) = \mathbb{R}^3$. However, not all vectors in $\mathbb{R}^3$ satisfy Equation (1).

3. 2.10

   Suppose $A = \{\mathbf{v}_1,\dots,\mathbf{v}_n\}$ is a set which contains a linearly dependent set of vectors. We may assume (possibly reindexing the vectors) that $\{\mathbf{v}_1,\dots,\mathbf{v}_m\}$ is a set of linearly dependent vectors, where $m \le n$. Then there are scalars $c_1,\dots,c_m$, not all zero, so that

   $$c_1\mathbf{v}_1 + \cdots + c_m\mathbf{v}_m = \mathbf{0}.$$

   Now we demonstrate a dependence with all the vectors in A. Let

   $$d_i = \begin{cases} c_i & 1 \le i \le m \\ 0 & m+1 \le i \le n \end{cases}$$

   Then not all the d_i are zero and

   $$\sum_{i=1}^{n} d_i\mathbf{v}_i = \sum_{i=1}^{m} c_i\mathbf{v}_i = 0.$$

   The analogous statement is "Any subset of a linearly independent set is linearly independent."