Stat 400 - Solutions to Last Homework

Section 7.3, Problem 33(c). A $100(1-\alpha)\%$ confidence interval for μ is given by

$$\left(\bar{x} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}},\ \bar{x} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\right)$$

where $\bar{x}$ is the sample mean, s is the sample standard deviation, and n is the size of the sample. You can look up the values $t_{\alpha/2,n-1}$ in Appendix A.5 in the back of the book.

For the data given in the problem, the sample mean is $\bar{x} = 438.29$ and the sample standard deviation is $s = 15.14$ (for anyone with MATLAB, these can be computed easily by putting the data into a vector `v` and using the commands `mean(v)` and `std(v)`). Since $\alpha = .05$, we have $\alpha/2 = .025$. Using Appendix A.5 gives $t_{.025,16} = 2.12$, and plugging all of this into the above formula gives the confidence interval

$$\left(438.29 - 2.12\frac{15.14}{\sqrt{17}},\ 438.29 + 2.12\frac{15.14}{\sqrt{17}}\right) = (430.51, 446.07).$$

The interval suggests that 440 is a plausible guess for the true mean, but that 450 is not.

Section 7.3, Problem 37(a). The Minitab output tells us that $\bar{x} = .9255$ and that $s = .0809$. Since $n = 20$, we have $s/\sqrt{n} = .0181$. As in the first part, we have $\alpha/2 = .025$, and Appendix A.5 gives $t_{.025,19} = 2.093$. Therefore the 95% confidence interval is given by

$$(.9255 - 2.093 \cdot .0181,\ .9255 - 2.093 \cdot .0181) = (.8876, .9634).$$

(b) A prediction interval with prediction level $100(1-\alpha)\%$ is given by

$$\left(\bar{x} - t_{\alpha/2,n-1}s\sqrt{1+1/n},\ \bar{x} + t_{\alpha/2,n-1}s\sqrt{1+1/n}\right)$$

where the numbers as the same as in part (a). Plugging them in gives a 95% prediction interval of (.752, 1.099).

Section 7.3, Problem 40(c). According to the proposition on page 292, a lower prediction bound with prediction level $100(1-\alpha)\%$ is given by

$$\left(\bar{x} - t_{\alpha,n-1}s\sqrt{1+1/n},\ \infty\right)$$

Since we want a 95% prediction interval, $\alpha = .05$. The data from Problem 17 of 7.2 gives us $\bar{x} = 125.39$, $s = 4.59$, and $n = 153$. Appendix A.5 doesn't give a value for $t_{.05,153}$, but we can use the value given for $t_{.05,120}$, which is 1.658, as a reasonable approximation. Plugging all of this in to the above formula gives a one-sided prediction interval of

$$\left(125.39 - 1.658 \cdot 4.59\sqrt{1+1/153},\ \infty\right) = (117.708,\ \infty).$$

Extra Problem. If μ is in the interval $(\bar{x} - t_{\alpha,n-1}s/\sqrt{n}, \infty)$, then we have

$$\bar{x} - t_{\alpha,n-1}s/\sqrt{n} < \mu$$

which, after subtracting μ from both sides, and adding $t_{\alpha,n-1}s/\sqrt{n}$ to both sides, gives us

$$\bar{x} - \mu < t_{\alpha,n-1}s/\sqrt{n}.$$

Dividing by the quantity $(s/\sqrt{n})$ gives

$$\frac{\bar{x} - \mu}{s/\sqrt{n}} < t_{\alpha,n-1}.$$

The left-hand side is exactly the random variable T described at the top of page 288, so we want to know

$$P(T < t_{\alpha,n-1})$$

but this is exactly $1 - \alpha$ by definition (see the box labeled "notation" on page 289).