Discrete Distributions

Recall that a Combination (unordered subset) is given by: $C_{b,a} = \binom{a}{b} = \frac{a!}{b!(a-b)!}$ and a Permutation (ordered subset) is given by $P_{b,a} = \frac{a!}{(a-b)!}$

Distribution	Experiment Type	Notation	Use When	pmf P(X=k)	*Valid for k :*	*Mean* E(X) = μ	*Variance* V(X) = σ^2	Special Case Of:
Bernoulli	Bernoulli (S or F)		Binary	$p^k q^{1-k}$	= 0, 1	p	$p \cdot q$	Binomial
Binomial		b(x; n, p) = $X \sim Bin(n, p)$	With replacement	$\binom{n}{k} \cdot p^k \cdot q^{n-k}$	= 0, 1, 2, … n	$n \cdot p$	$n \cdot p \cdot q$	Multinomial: k = 2
Hypergeometric		h(x; n, M, N)	Random Sample Without replacement	$\frac{\binom{M}{k}\binom{L}{n-k}}{\binom{N}{n}}$	≤ n and M AND ≥ 0 and n-L	$n \cdot \frac{M}{N}$	$n \cdot \frac{M}{N} \cdot \left(\frac{N-n}{N-1}\right) \cdot \left(1 - \frac{M}{N}\right)$ $= n \cdot p \cdot q \cdot \left(\frac{N-n}{N-1}\right)$	
Negative Binomial	Bernoulli (S or F)	nb(x; r, p)	Trials independent, P(S) same for each, go until **r** successes	$\binom{k+r-1}{r-1} \cdot p^r \cdot q^k$	= 0, 1, 2, … ∞	$r \cdot \frac{q}{p}$	$r \cdot \frac{q}{p^2}$	
Geometric	Bernoulli (S or F)	nb(x; 1, p)	r = 1 above	$p \cdot q^k$	= 0, 1,b 2 … ∞	$\frac{q}{p}$	$\frac{q}{p^2}$	Negative Binomial: r = 1
Poisson	None	p(x; λ) = $X \sim P(\lambda)$	Binomial has n > 50 AND $\lambda = n \cdot p < 5$	$\frac{e^{-\lambda} \cdot \lambda^k}{k!}$	= 0, 1, 2 … ∞	λ	λ	
Multinomial		p(x_1, …, x_k)		$\frac{n!}{x_1! \cdot x_2! \ldots x_k!} \cdot p_1^x \cdot \ldots p_k^x$	= 0, 1, 2 … ∞ $x_1 + \ldots x_k = n$			

p = the probability of a success (S)

q = the probability of a failure (F) = 1- p

r = total # of successes *needed* to end the experiment

n = total # of *actual* outcomes (# of trials)

k = total # of *actual* successes

n – k = total # of *actual* failures

N = total # of *possible* outcomes = M + L

M = total # of *possible* successes (S)

L = total # of *possible* failures (F)

$\frac{N-n}{N-1}$ = "Finite Population Correction Factor". If N>>n, this is ≈ 1 and V(X) = npq

λ = either $\begin{cases} a \quad given, \quad positive \quad const. \quad (Poisson.Distribution) \\ f(t) = \lambda(t) = \alpha \cdot t \quad (Poisson.Process) \end{cases}$

α = λ(1) = average # of observations in unit time (rate)

p = M/N

q = 1 – M/N

k + r -1 = = "waiting time"

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Continuous Distributions

Distribution	Notes	Notation	Use When	pmf, pdf P(X)	cdf = P(X ≤ x)	*Mean* E(X) = μ	*Variance* V(X) = σ^2	Special Case Of:
Uniform		f(x) = $X \sim U(A,B)$		$\begin{cases} \frac{1}{B-A} & A \le x \le B \\ 0 & otherwise \end{cases}$	$F(x) = \int_{-\infty}^{x} f(t)dt$	$\frac{A+B}{2}$	$\frac{(B-A)^2}{12}$	
Gamma	α > 0 β > 0	f(x; α, β) = f(x)		$\begin{cases} \frac{1}{\beta^\alpha \cdot \Gamma(\alpha)} \cdot x^{\alpha-1} \cdot e^{-\frac{x}{\beta}} & x \ge 0 \\ 0 & otherwise \end{cases}$	$F(x;\alpha,\beta) = F\left(\frac{x}{\beta};\alpha\right)$	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$	
Standard Gamma		f(x; α, 1)	use table A.4	$\begin{cases} \frac{1}{\Gamma(\alpha)} \cdot x^{\alpha-1} \cdot e^{-x} & x \ge 0 \\ 0 & otherwise \end{cases}$	$F(x;\alpha) = \int_0^x f(t;\alpha,1)dt$ = the "incomplete gamma function"	α	α	Gamma: β = 1
Exponential	λ > 0 memoryless	f(x; λ)		$\begin{cases} \lambda \cdot e^{-\lambda \cdot x} & x \ge 0 \\ 0 & otherwise \end{cases}$	$F(x) = \begin{cases} 1-e^{-\lambda \cdot x} & x \ge 0 \\ 0 & x < 0 \end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	Gamma: α = 1 β = 1/λ
r - Erlang	r > 0 Continuous analog of neg. binom.	$f_r(t; \lambda, r)$		$\begin{cases} \frac{\lambda^r \cdot t^{r-1}}{(r-1)!} \cdot e^{-\lambda \cdot t} & t \ge 0 \\ 0 & otherwise \end{cases}$	$F_r(t) = \begin{cases} 1-\sum_{n=0}^{r-1}\left(\frac{(\lambda \cdot t)^n}{n!}\right) \cdot e^{-\lambda \cdot t} & t > 0 \\ 0 & t \le 0 \end{cases}$	$\frac{r}{\lambda}$	$\frac{r}{\lambda^2}$	Gamma: α = r β = 1/λ
Chi-Squared	ν = degrees of freedom (+ real #)	f(x; ν) = $X \sim \chi^2(\upsilon)$		$\begin{cases} \frac{x^{\left(\frac{\upsilon}{2}\right)-1}}{2^{\frac{\upsilon}{2}} \cdot \Gamma\left(\frac{\upsilon}{2}\right)} \cdot e^{-\frac{x}{2}} & x \ge 0 \\ 0 & otherwise \end{cases}$		ν	2ν	Gamma: α = ν/2 β = 2
Normal	"Bell Curve" $-\infty < x < \infty$ "Gaussian"	f(x; μ, σ) = $X \sim N(\mu,\sigma^2)$	use table	$\frac{1}{\sqrt{2 \cdot \pi} \cdot \sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$\Phi(z) = F(z) = \int_{-\infty}^{z} f(x)dx$	μ	σ^2	
Standard Normal		f(z; 0, 1) = $X \sim N(0,1)$	use table A.3	$\frac{1}{\sqrt{2\pi}} \cdot e^{-\left(\frac{x^2}{2}\right)}$	$\Phi(z) = P(Z \le z)$	0	1	Normal: μ = 0 σ = 1

Note: the gamma function is $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} \cdot e^{-x} dx$ and:

1. $\Gamma(\alpha) = (\alpha-1) \cdot \Gamma(\alpha-1)$ for α > 1
2. $\Gamma(n) = (n-1)!$ for any integer n > 0
3. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

β = "Scale Parameter": stretches or compresses pdf in x

$P(a \le X \le b) = F(b) - F(a)$

$P(X \ge a) = 1 - F(a)$

50th percentile = Median = $\tilde{\mu}$

$F(\tilde{\mu}) = cdf(\tilde{\mu}) = 0.5$

$E(X) = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$

$E[h(X)] = \int_{-\infty}^{\infty} h(x) \cdot f(x) \cdot dx$

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