Stat 400 **FINAL EXAM** **Dec. 20, 2005**
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1. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} 5x^4, \ 0 \leq x \leq 1, \\ 0, otherwise. \end{cases}$$

(a) Find $E(X)$.

(b) Find $E(X^2)$.

(c) Find $V(X)$.

(d) Find $F(x)$, the cumulative distribution function of X.

(e) Find the median of X.

(20 points)

2. This was Problem 3 (the component problem on your Midterm 1 - that is fall 2006 Midterm 1).

3. Suppose X and Y are random variables defined on the same sample space with the following joint probability mass function.

X \ Y	0	1	2	3
0	1/32	3/32	3/32	1/32
1	2/32	6/32	6/32	2/32
2	1/32	3/32	3/32	1/32

(a) Compute the probability mass functions of the random variables X and Y. Each has binomial distribution, what are the binomial parameters n and p for X and Y?

(b) Are X and Y independent?

(c) Compute the probability mass function of the random variable $W = X + Y$.

(d) Compute the probability mass function of the random variable $W = XY$.

(e) What is $Cov(X, Y)$ (use your answer from (b) to avoid making a long computation).

(20 points)

4. Let $X_1, X_2, \cdots, X_n$ be a random sample from an exponential distribution with parameter $\lambda = \frac{1}{2}$. Let $W = X_1 + X_2 + \cdots + X_n$.

(a) Find the moment generating function $M_W(t)$ of W.

(Hint: all the moment generating functions $M_{X_i}(t)$'s are equal to the moment generating function of an exponential random variable with parameter $\frac{1}{2}$. Now multiply all the $M_{X_i}(t)$'s together to get $M_W(t)$.)

(b) This moment generating function (i.e. the moment generating function of W)is the moment generating function of one of the standard distributions on the handout on distributions-which one?

(20 points)

5. Let $x_1, x_2, \cdots, x_n$ be a sample from a Bernoulli distribution with parameter p (so each x_i is either 1 or 0). Let $\hat{p}$ be the sample proportion (i.e. the number of 1's divided by n or equivalently the number of observed successes divided by the number of observations) and $\hat{q} = 1 - \hat{p}$. In formula (7.11) of the text it is stated that the interval

$$(\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}, \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}})$$

is the observed value of an approximate $100(1-\alpha)\%$ confidence interval for the population proportion (success probability) p.

A study of 100 football helmets of a certain type found that 10 showed damage when subjected to an impact test. Let p denote that true proportion of football helmets of this type that would be damaged by the impact test.

Use the above formula to construct a 90% confidence interval for p.
(Hint: think of the 100 helmets as a sample of size 100 from the population of all footballs helmets of the given type.)

(10 points)

6. Let $X_1, X_2, \cdots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2. We wish to predict the next observation X_{n+1}. Assume that we know σ^2. Let $\overline{X} = \frac{X_1+\cdots+X_n}{n}$ be the sample mean for the first n observations. Use the theorem that $Z = \frac{\overline{X}-X_{n+1}}{\sqrt{\frac{n+1}{n}}\sigma}$ has standard normal distribution and prove that the random interval

$$(-\infty,\ \overline{X} + z_\alpha \sqrt{\frac{n+1}{n}}\sigma)$$

is a $100(1-\alpha)\%$ prediction interval for the next observation X_{n+1}.

(20 points)