Solutions of Practice Midterm 2

1 (a)

x	0	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

x	0	1	2
$P(Y=y)$	$\frac{1}{4}$	$3/8$	$3/8$

(b) No

(c)

w	0	1	2	3	4
$P(W=w)$	$\frac{1}{16}$	$3/16$	$7/16$	$3/16$	$2/16$

(d) $Cov(X,Y) = E(XY) - E(X)E(Y)$

$= \frac{9}{8} - (1)(\frac{9}{8}) = 0$

(e) $\rho_{X,Y} = 0$

2. (a) $L(\theta) = \frac{1}{\theta^2} x_1 e^{-x_1/\theta} \cdots \frac{1}{\theta^2} x_n e^{-x_n/\theta}$

$$L(\theta) = \frac{1}{\theta^{2n}} (x_1 \cdots x_n) e^{-\frac{x_1 + \cdots + x_n}{\theta}}$$

Put $h(\theta) = \ln L(\theta)$

$$= -2n \ln\theta + \ln(x_1 \cdots x_n) - \frac{x_1 + \cdots + x_n}{\theta}$$

$$h'(\theta) = -\frac{2n}{\theta} + \frac{x_1 + \cdots + x_n}{\theta^2}$$

so $h'(\theta) = 0 \iff \theta = \frac{x_1 + \cdots + x_n}{2n} = \frac{\bar{x}}{2}$

(b) $\hat{\theta} = \frac{\bar{X}}{2}$

$E(\bar{X})$ is always the population mean so $E(\bar{X}) = 2\theta$

so $E\left(\frac{\bar{X}}{2}\right) = \frac{1}{2} E(\bar{X}) = \frac{1}{2}(2\theta) = \theta$

Let $Y \sim N(50, 100)$.

Then $P(X > 60) \approx P(Y > 60)$

$$P(Y > 60) = P\left(\frac{Y - 50}{10} > \frac{60 - 50}{10}\right)$$

$$= P(Z \geq 1)$$

$$= 1 - P(Z \leq 1)$$

$$= 1 - .8413$$

$$= .1587$$