TEST 1

1. Let X be a discrete random variable with probability mass function p given by

$$p(-1) = 1/8, p(0) = 3/8, p(1) = 3/8, p(2) = 1/8.$$

(1) Find $E(X)$.
(2) Find $V(X)$.
(3) Find $F(x)$, the cumulative distribution function of X.

(15 points)

2. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} (n+1)x^n, 0 \leq x \leq 1, \\ 0, \text{ otherwise.} \end{cases}$$

(1) Find $E(X)$.
(2) Find $V(X)$.
(3) Find $F(x)$, the cumulative distribution function of X.
(4) Find the median of X.
(5) Find the 75-th percentile of X.

(Your answers 1., 2., 4,,5, should be functions of n. Your answer to 3. should be a function of x and n.)

(25 points)

3. A population starts with one member at time $t = 1$. It either divides in two with probability p or dies with probability $1 - p$. If it divides, then both of its children behave independently with the same alternatives at time $t = 2$. Their children again behave independently with the same alternatives (divide or die) at time $t = 3$. Let X be the number of members of the population at time $t = 3$. Find the probability distribution of the random variable X_3 - this means find the possible values of X_3 and the probabilities of those values.

(10 points)

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