

1. Roll a fair die. Let  $X$  be the number that appears.
- (a) Find the probability mass function of  $X$  (i.e. what values does  $X$  take and with what probabilities - make a table).
  - (b) Find  $E(X)$  (hint: the sum of the whole numbers from 1 to 6 is 21).
  - (c) Find  $E(X^2)$  (hint: the sum of the whole numbers squared from 1 to 6 is 91).
  - (d) Find  $V(X)$ .
  - (e) Find  $F(x)$ , the cumulative distribution function of  $X$  (you don't have to draw the graph, just give the formulas).
  - (f) Make the change of random variable  $Y = X - 1$ . Compute the probability mass function of  $Y$ .

(18 points)

2. Let  $X$  be a continuous random variable with probability density function

$$f(x) = \begin{cases} 2(1-x), & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find  $E(X)$ .
- (b) Find  $V(X)$ .
- (c) Find  $F(x)$ , the cumulative distribution function of  $X$ .
- (d) Find  $\tilde{\mu}$ , the median of  $X$ .

(12 points)

3. Suppose 5 cards of a poker hand are dealt one at a time without replacement. If the first two cards drawn are aces what is the probability two of the last three will be aces.

(8 points)

4. Suppose that each component in the following system has probability  $p$  of working. What is the probability the system will work?

(12 points)

